

Higher Still - 2003 / 2004

MATHEMATICS

Advanced Higher Grade

EXTENDED UNIT TEST - UNIT 1

Time allowed - 50 minutes

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. **Calculators may be used.**
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This Unit Test contains questions graded at all levels.**

All questions should be attempted

1. Find the term independent of x in the expansion of

$$(1+x^2)\left(3x+\frac{1}{x}\right)^{10}. \quad (4)$$

2. Show that the derivative of the function defined by $f(x) = \ln(\tan 3x)$ can be written as:

$$f'(x) = 6 \operatorname{cosec} 6x. \quad (4)$$

3. Find all the possible solutions to the following system of equations:

$$\begin{aligned} 5x - 3y + 4z &= 6 \\ 13x - 7y - 12z &= 14 \\ -7x + 4y &= -8 \end{aligned} \quad (5)$$

4. Show that the improper fraction $\frac{4x^3 + 9x^2 + 17x - 9}{2x^3 + 5x^2 - 3x}$ can be written as $A + \frac{B}{x} + \frac{C}{x+3} + \frac{D}{2x-1}$, where A, B, C and D are real numbers.

State the values of A, B, C and D . (5)

5. A particle travels in a straight line so that its displacement, x metres, from a fixed point O after a time t is given by $x = \cos t + \sqrt{3} \sin t$.

Find : (a) The displacement from O when it comes instantaneously to rest; (3)

(b) its acceleration at this moment; (2)

(c) its maximum velocity. (1)

6. Use the substitution $x = \tan \theta$ to show that:

$$\int_0^1 \frac{4}{(1+x^2)^{\frac{3}{2}}} dx = 2\sqrt{2}. \quad (5)$$

7. The function f is defined by $f(x) = x^3 e^{-x}$.

(a) Find the stationary points and determine their nature. (5)

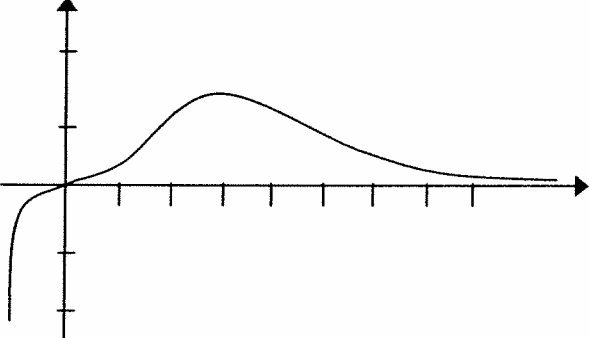
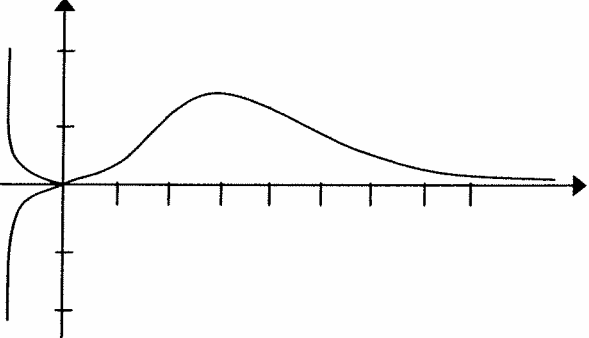
(b) Sketch the graph of $y = f(x)$. (2)

(c) On the same diagram sketch the graph of $y = |f(x)|$. (1)

END OF QUESTION PAPER

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	<p>ans: $3^4 \times 966$</p> <p style="text-align: right;">4 marks</p> <ul style="list-style-type: none"> • 1 using Binomial expansion • 2 identifying appropriate terms • 3 multiplying brackets • 4 stating answer 	<ul style="list-style-type: none"> • 1 ${}^{10}C_5(3x)^5\left(\frac{1}{x}\right)^5 = 252 \times 3^5$ • 2 ${}^{10}C_6(3x)^4\left(\frac{1}{x}\right)^6 = 210 \times \frac{3^4}{x^2}$ • 3 $(1+x^2)(252 \times 3^5) = 252 \times 3^5 + 252 \times 3^5 x^2$ $(1+x^2)\left(210 \times \frac{3^4}{x^2}\right) = 210 \times \frac{3^4}{x^2} + 210 \times 3^4$ 4 $252 \times 3^5 + 210 \times 3^4 = 3^4 \times 966$
2.	<p>ans: Proof</p> <p style="text-align: right;">4 marks</p> <ul style="list-style-type: none"> • 1 differentiating ln... • 2 knowing to use the chain rule • 3 knowing to use trig identities • 4 completing proof 	<p>1 $\frac{1}{\tan 3x} \times \frac{d}{dx} \tan 3x$</p> <p>2 $\frac{1}{\tan 3x} \times 3 \sec^2 3x$</p> <p>3 $\frac{\cos 3x}{\sin 3x} \times \frac{3}{\cos^2 3x} = \frac{3}{\sin 3x \cos 3x}$</p> <p>4 $\frac{3}{\sin 3x \cos 3x} = \frac{3}{\frac{1}{2} \sin 6x} = 6 \operatorname{cosec} 6x$</p>
3.	<p>ans: $(16t, 28t - 2, t)$</p> <p style="text-align: right;">5 marks</p> <ul style="list-style-type: none"> • 1 using augmented matrix • 2 first modified system • 3 second modified system • 4 setting $z =$ parameter & finding y in terms of parameter • 5 finding x in terms of parameter 	<p>1 $\begin{pmatrix} 5 & -3 & 4 & 6 \\ 13 & -7 & -12 & 14 \\ -7 & 4 & 0 & -8 \end{pmatrix}$</p> <p>2 $\begin{pmatrix} 5 & -3 & 4 & 6 \\ 0 & -4 & 112 & 8 \\ 0 & -1 & 28 & 2 \end{pmatrix}$</p> <p>3 $\begin{pmatrix} 5 & -3 & 4 & 6 \\ 0 & 1 & -28 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$</p> <p>4 $z = t \quad y = 28t - 2$</p> <p>5 $x = 16t$</p>

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4.	<p>ans: $A = 3, B = -4, C = 1$</p> <p style="text-align: right;">5 marks</p> <ul style="list-style-type: none"> • 1 knowing to use long division to restate the expression • 2 restating expression correctly • 3 knowing to express the fraction as a sum of fractions • 4 knowing to find values for constants • 5 correctly calculating constants 	<ul style="list-style-type: none"> • 1 $2 + \frac{-x^2 + 22x - 9}{2x^3 + 5x^2 - 3x}$ • 2 $2 + \frac{-x^2 + 22x - 9}{x(x+3)(2x-1)}$ • 3 $\frac{-x^2 + 22x - 9}{x(x+3)(2x-1)} = \frac{B}{x} + \frac{C}{x+3} + \frac{D}{2x-1}$ • 4 $B(x+3)(2x-1) + Cx(2x-1) + Dx(x+3) = -x^2 + 22x - 9$ • 5 answer
5(a)	<p>ans: 2 metres</p> <p style="text-align: right;">3 marks</p> <ul style="list-style-type: none"> • 1 knowing how to calculate v • 2 equating $v = 0$ and solve for t • 3 finding displacement 	<ul style="list-style-type: none"> • 1 $v = \frac{dx}{dt} = -\sin t + \sqrt{3} \cos t$ $-\sin t + \sqrt{3} \cos t = 0$ • 2 $\Rightarrow 2 \cos\left(t + \frac{\pi}{6}\right) = 0$ $\Rightarrow t = \frac{\pi}{3}$ • 3 $t = \frac{\pi}{3} \Rightarrow x = 2$
(b)	<p>ans: -2 ms^{-2}</p> <p style="text-align: right;">2 marks</p> <ul style="list-style-type: none"> • 1 knowing how to calculate a • 2 evaluating a 	<ul style="list-style-type: none"> • 1 $a = \frac{dv}{dt} = -2 \sin\left(t + \frac{\pi}{6}\right)$ • 2 $a = -2 \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$
(c)	<p>ans: 2 ms^{-1}</p> <p style="text-align: right;">1 mark</p>	<ul style="list-style-type: none"> • 1 answer
6.	<p>ans: proof</p> <p style="text-align: right;">5 marks</p> <ul style="list-style-type: none"> • 1 dealing with substitution • 2 finding dx and new limits • 3 simplifying expression • 4 integrating correctly • 5 evaluating correctly 	<ul style="list-style-type: none"> • 1 $(1 + x^2)^{\frac{3}{2}} = \sec^3 \theta$ • 2 $dx = \sec^2 \theta d\theta$, limits = $0, \frac{\pi}{4}$ • 3 $\int_0^{\frac{\pi}{4}} 4 \cos \theta d\theta$ • 4 $[4 \sin \theta]_0^{\frac{\pi}{4}}$ • 5 answer

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7(a)	<p>ans: (0, 0) Pt of inflexion (3, 1.344) Max TP</p> <p style="text-align: right;">5 marks</p> <ul style="list-style-type: none"> • 1 differentiating using product rule • 2 equating to zero • 3 solving for x • 4 evaluating y coordinates • 5 justifying nature 	<ul style="list-style-type: none"> • 1 $\frac{dy}{dx} = 3x^2e^{-x} - x^3e^{-x}$ • 2 $x^2e^{-x}(3-x) = 0$ 3 $x = 0, x = 3$ 4 $y = 0, y = 1.344$ 5 from $\frac{d^2y}{dx^2}$ or nature table
(b)	<p>ans: sketch</p> <p style="text-align: right;">2 marks</p> <ul style="list-style-type: none"> • 1 sketch showing all relevant points and turning points • 2 showing how curve approaches asymptote 	
(c)	<p>ans: sketch</p> <p style="text-align: right;">1 mark</p> <ul style="list-style-type: none"> • 1 reflecting negative parts of $y = f(x)$ in the x-axis 	

Total : 37 marks