

Cfe AH Mathematics**Unit 1 Practice Course Assessment Paper 1**

(1) Differentiate with respect to x :

(a) $y = \sin\left(\frac{1+x^2}{1-x}\right)$ (3)

(b) $y = e^{\sqrt{x}} \ln(2+x^2)$ (3)

(c) $y = 10^{\sin x}$ (3)

(2) (a) Obtain the partial fractions for $\frac{x^2 - 2x}{(x+2)(x^2 + 4)}$. (3)

(b) Hence find $\int \frac{x^2 - 2x}{(x+2)(x^2 + 4)} dx$. (3)

(3) An implicit function is defined by:- $y^2 - 2xy = 2x$.

(a) Show that $\frac{dy}{dx} = \frac{y+1}{y-x}$. (3)

(b) Hence, find an expression for $\frac{d^2y}{dx^2}$. (6)

(4) For the curve with parametric equations:-

$$x = 2\theta + \sin 2\theta, y = \cos 2\theta,$$

(a) prove that $\frac{dy}{dx} = -\tan\theta$ (3)

(b) prove that $\frac{d^2y}{dx^2} = -\frac{1}{4}\sec^4\theta$. (3)

(5) Use the substitution, $x = 2\sin\theta$, to show that:-

$$\int_0^1 \frac{1}{(4-x^2)^{\frac{3}{2}}} dx = \frac{\sqrt{3}}{12} \quad (5)$$

(6) (a) Using integration by parts and partial fractions to show that:-

$$\int \frac{\ln x}{(1+x)^2} = \ln\left(\frac{x}{1+x}\right) - \frac{\ln x}{(1+x)} + c. \quad (5)$$

(b) Hence prove that $\int_{e^{-1}}^e \frac{\ln x}{(1+x)^2} dx = 0. \quad (5)$

(7) Obtain the general solution of the differential equation

$$4\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 9y = 7\sin x + 17\cos x.$$

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Hence find the particular solution for which $y = 3$ and $\frac{dy}{dx} = \frac{5}{2}$ when $x = 0. \quad 3$

Mark Scheme
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<p>① a</p> <p>ans : $\frac{1+2x-x^2}{(1-x)^2} \cos\left(\frac{1+x^2}{1-x}\right)$</p> <ul style="list-style-type: none"> •1 knows to use the chain rule •2 knows to use the quotient rule •3 completes simplification <p>3 marks</p>	<ul style="list-style-type: none"> •1 $\frac{d}{dx}\left(\frac{1+x^2}{1-x}\right) \cos\left(\frac{1+x^2}{1-x}\right)$ •2 $\frac{2x(1-x)+(1+x^2)}{(1-x)^2}$ •3 $\frac{1+2x-x^2}{(1-x)^2} \cos\left(\frac{1+x^2}{1-x}\right)$
<p>① b</p> <p>ans : $\frac{1}{2\sqrt{x}} e^{\sqrt{x}} \left[\ln(2+x^2) + \frac{4x^{\frac{3}{2}}}{2+x^2} \right]$</p> <ul style="list-style-type: none"> •1 knows to use the product rule •2 differentiates $e^{\sqrt{x}}$ •3 differentiates $\ln(2+x^2)$ <p>3 marks</p>	<ul style="list-style-type: none"> •1 $\frac{1}{2} x^{-\frac{1}{2}} e^{\sqrt{x}} \ln(2+x^2) + e^{\sqrt{x}} \frac{2x}{2+x^2}$ •2 $\frac{1}{2} x^{-\frac{1}{2}} e^{\sqrt{x}}$ •3 $\frac{2x}{2+x^2}$
<p>① c</p> <p>ans : $y = 10^{\sin x} \cos x \ln 10$</p> <ul style="list-style-type: none"> •1 knows to use logarithmic differentiation •2 knows to differentiate implicitly •3 knows to replace y and finds $\frac{dy}{dx}$ <p>3 marks</p>	<ul style="list-style-type: none"> •1 $\ln y = \ln 10^{\sin x} \Rightarrow \ln y = \sin x \ln 10$ •2 $\frac{1}{y} \frac{dy}{dx} = \cos x \ln 10$ •3 $y = 10^{\sin x} \cos x \ln 10$

(2)

a ans : $\frac{1}{x+2} - \frac{2}{x^2+4}$

- .1 knows how to find partial fractions
- .2 knows how to find A, B and C
- .3 finds A, B and C

3 marks

b

ans: $\ln(x+2) - \tan^{-1}\left(\frac{x}{2}\right) + c$

- .1 knows to express integral in P.F's.
- .2 knows how to integrate $\frac{1}{x+2}$
- .3 knows how to integrate $\frac{2}{x^2+4}$

3 marks

.1 $\frac{x^2 - 2x}{(x+2)(x^2+4)} = \frac{A}{x+2} - \frac{Bx+C}{x^2+4}$

.2 $x^2 - 2x = A(x^2 + 4) + (Bx + C)(x + 2)$

.3 $A = 1, B = 0, C = -2$

.1 $\int \left(\frac{1}{x+2} - \frac{2}{x^2+4} \right) dx$

.2 $\ln(x+2)$

.3 $-\tan^{-1}\left(\frac{x}{2}\right) (+c)$

(3)

a ans : $\left(\frac{y+1}{y-x} \right),$

- .1 knows how to differentiate y^2
- .2 knows how to differentiate $2xy$
- .3 simplifies expression to read $\frac{dy}{dx}$

3 marks

.1 $2y \frac{dy}{dx}$

.2 $2y + 2x \frac{dy}{dx}$

.3 $\left(\frac{y+1}{y-x} \right)$

b

ans: $\frac{(y+1)(y-2x-1)}{(y-x)^3}$

- .1 knows how to find $\frac{d^2y}{dx^2}$
- .2 knows how to differentiate $2y \frac{dy}{dx}$
- .3 knows how to differentiate $2x \frac{dy}{dx}$
- .4 finds $\frac{d^2y}{dx^2}$ in terms of $\frac{dy}{dx}$
- .5 replaces $\frac{dy}{dx}$ by $\left(\frac{y+1}{y-x} \right)$
- .6 simplifies $\frac{d^2y}{dx^2}$

6 marks

.1 $\frac{d}{dx} \left(2y \frac{dy}{dx} - 2y - 2x \frac{dy}{dx} = 2 \right)$

.2 $2 \left(\frac{dy}{dx} \right)^2 + 2y \frac{d^2y}{dx^2}$

.3 $-2 \frac{dy}{dx} - 2x \frac{d^2y}{dx^2}$

.4 $(y-x) \frac{d^2y}{dx^2} = 2 \frac{dy}{dx} - \left(\frac{dy}{dx} \right)^2$

.5 $(y-x) \frac{d^2y}{dx^2} = 2 \left(\frac{y+1}{y-x} \right) - \left(\frac{y+1}{y-x} \right)^2$

.6 $\frac{(y+1)(y-2x-1)}{(y-x)^3}$

<p>(4) a</p> <p>ans : Proof</p> <ul style="list-style-type: none"> •¹ know to differentiate x and y w.r.t θ •² know how to find $\frac{dy}{dx}$ •³ completes the proof <p>3 marks</p>	<ul style="list-style-type: none"> •¹ $\frac{dx}{d\theta} = 2 + 2\cos 2\theta$ and $\frac{dy}{d\theta} = -2\sin 2\theta$ •² $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2\sin 2\theta}{2 + 2\cos 2\theta}$ •³ $\frac{dy}{dx} = -\tan \theta$
<p>(4) b</p> <p>ans: Proof</p> <ul style="list-style-type: none"> •¹ knows how to find $\frac{d^2y}{dx^2}$ •² know the derivative of $\tan \theta$ •³ completes the proof <p>3 marks</p>	<ul style="list-style-type: none"> •¹ $\frac{d^2y}{dx^2} = \frac{d}{d\theta} \left(\frac{dy}{dx} \right) \frac{d\theta}{dx} = \frac{d}{d\theta} (-\tan \theta) \frac{d\theta}{dx}$ •² $\frac{d}{d\theta} (-\tan \theta) = -\sec^2 \theta$ •³ $\frac{d^2y}{dx^2} = -\frac{1}{4} \sec^4 \theta$

<p>(5)</p> <p>ans : Proof</p> <ul style="list-style-type: none"> •¹ deals with the substitution •² copes with dx and limits •³ simplifies substitutions •⁴ integrates correctly •⁵ Evaluates correctly <p>5 marks</p>	<ul style="list-style-type: none"> •¹ $(4 - 4\sin^2 \theta)^{\frac{1}{2}} = (4\cos^2 \theta)^{\frac{1}{2}} = 2\cos \theta$ •² $dx = 2\cos \theta d\theta$, limits = 0 and $\pi/6$ •³ $\frac{2\cos \theta}{8\cos^3 \theta} = \frac{1}{4}\sec^2 \theta$ •⁴ $\left[\frac{1}{4}\tan \theta \right]_0^{\pi/6}$ •⁵ $\frac{1}{4} \times \frac{1}{\sqrt{3}} = \frac{1}{4\sqrt{3}} = \frac{\sqrt{3}}{12}$
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<p>(b) a</p> <p>ans : Proof</p> <ul style="list-style-type: none"> •¹ knows how to integrate by parts •² know how to use partial fractions •³ finds A and B •⁴ knows how to integrate $\frac{1}{x} - \frac{1}{x+1}$ •⁵ simplifies ln's and completes proof <p>5 marks</p>	<ul style="list-style-type: none"> •¹ $-\frac{\ln x}{(1+x)} + \int \frac{dx}{x(x+1)}$ •² $\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$ •³ A = 1, B = -1 •⁴ $\ln x - \ln(x+1)$ •⁵ $-\frac{\ln x}{(1+x)} + \ln\left(\frac{x}{x+1}\right) + c$
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contd....

Qu	Marking Scheme Give 1 mark for each •	Illustration of evidence for awarding a mark for each •
<p>(b) b</p> <p>ans : Proof</p> <ul style="list-style-type: none"> •¹ knows how to evaluate a definite integral •² substitutes correctly •³ evaluates $\left[\ln\left(\frac{e}{e+1}\right) - \frac{\ln e}{(e+1)} \right]$ •⁴ evaluates $\left[\ln\left(\frac{e^{-1}}{e^{-1}+1}\right) - \frac{\ln e^{-1}}{(e^{-1}+1)} \right]$ •⁵ completes proof <p>5 marks</p>	<ul style="list-style-type: none"> •¹ $\left[\ln\left(\frac{x}{x+1}\right) - \frac{\ln x}{(x+1)} \right]_e^{-1}$ •² $\left[\ln\left(\frac{e}{e+1}\right) - \frac{\ln e}{(e+1)} \right]$ and $\left[\ln\left(\frac{e^{-1}}{e^{-1}+1}\right) - \frac{\ln e^{-1}}{(e^{-1}+1)} \right]$ •³ $1 - \ln(e+1) - \frac{1}{e+1}$ •⁴ $-\ln(e+1) + \frac{e}{e+1}$ •⁵ $\left(1 - \ln(e+1) - \frac{1}{e+1}\right) - \left(-\ln(e+1) + \frac{e}{e+1}\right)$ $= 1 - \frac{1}{e+1} - \frac{e}{e+1} = \frac{e+1-1-e}{e+1} = 0$ 	

	Give one mark for each •	Illustrations for awarding each mark
7	<p>ans: $y = Ae^{\frac{3}{2}x} + Bxe^{\frac{3}{2}x} - \sin x + \cos x$</p> <p style="text-align: center;">7 marks</p> <ul style="list-style-type: none"> • correct auxiliary equation • solves auxiliary equation correctly • correct complementary function • correct form of particular integral • continues correctly • correct particular integral • correct general solution <p>ans: $y = 2e^{\frac{3}{2}x} + \frac{1}{2}xe^{\frac{3}{2}x} - \sin x + \cos x$</p> <p style="text-align: center;">3 marks</p> <ul style="list-style-type: none"> • correct value for A • correct derivative • correct value for B 	<p>7 marks</p> <ul style="list-style-type: none"> • $4m^2 - 12m + 9 = 0$ • $m = \frac{3}{2}$ (twice) • $y = Ae^{\frac{3}{2}x} + Bxe^{\frac{3}{2}x}$ • $y = C \sin x + D \cos x$ • $C = -1$ or $D = 1$ • $y = -\sin x + \cos x$ • $y = Ae^{\frac{3}{2}x} + Bxe^{\frac{3}{2}x} - \sin x + \cos x$ <p>3 marks</p> <ul style="list-style-type: none"> • $A = 2$ • $\frac{dy}{dx} = 3e^{\frac{3}{2}x} + Be^{\frac{3}{2}x} + \frac{3}{2}Bxe^{\frac{3}{2}x} - \cos x - \sin x$ • $B = \frac{1}{2}$