Established 1791

FORTROSE ACADEMY

Mathematics
Higher Prelim Examination 2009/2010
Assessing Units 1 & 2
Paper 2

NATIONAL QUALIFICATIONS

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

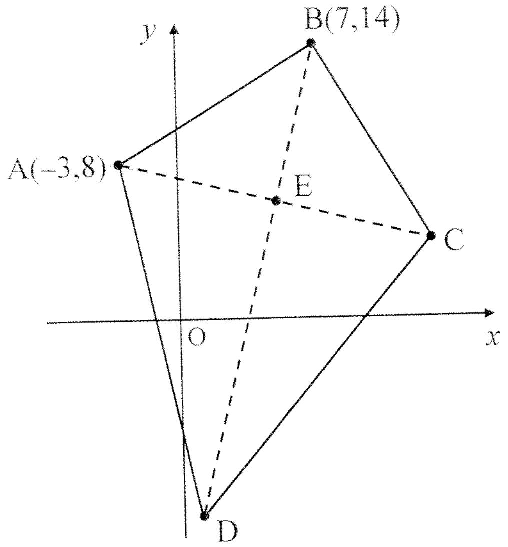
The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

ALL questions should be attempted

1. Kite ABCD has two of its vertices at A(−3,8) and B(7,14) as shown.



(a) Given that the equation of the longer diagonal BD is $y = 4x - 14$, find the equation of the short diagonal AC expressing your answer in the form $ax + by + c = 0$ and write down the values of a, b and c. 4

(b) Find the coordinates of E, the point of intersection of the two diagonals. 3

(c) Hence establish the coordinates of C. 2

2. Two functions are defined on suitable domains as $f(x) = 4x + 1$ and $g(x) = \dfrac{1}{x-1}$.

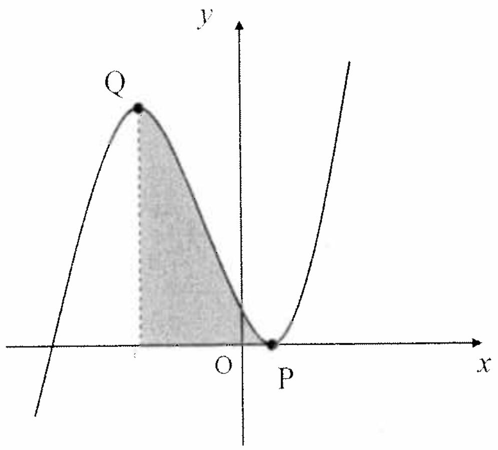
(a) If $h(x) = f(g(x))$, show clearly that $h(x)$ can be written as

$$h(x) = \frac{x+3}{x-1}.$$ 3

(b) Show that value of $h(\sqrt{5})$ can be expressed in the form $p + \sqrt{5}$ and write down the value of p. 4

3. A function is defined on the set of real numbers as $f(x) = 2x^3 + 3x^2 - 12x + 7$.

Part of the graph of $y = f(x)$ is shown below.



(a) Find the coordinates of the stationary points P and Q. 5

(b) Calculate the shaded area in the diagram. 4

4. Solve algebraically the equation

$$15\sin 2x^\circ = 10\cos x^\circ \qquad \text{for} \qquad 0 \leq x < 360.$$ 5

5. A recurrence relationship is such that $U_{n+1} = \frac{a}{4}U_n + 12$.

(a) If $U_0 = 16$ show clearly that $U_2 = a^2 + 3a + 12$. 2

(b) Hence find a if $U_2 = 30$ and $a > 0$. 3

(c) Explain why this sequence has a limit and find the limit. 3

6. Diagram 1 shows a rectangular plate of transparent plastic moulded into a parabolic shape and pegged to the ground to form a cover for growing plants. Triangular metal frames are placed over the cover to support it and prevent it blowing away in the wind.
Diagram 2 shows an end view of the cover and the triangular frame related to the origin O and axes Ox and Oy. (All dimensions are given in centimetres.)

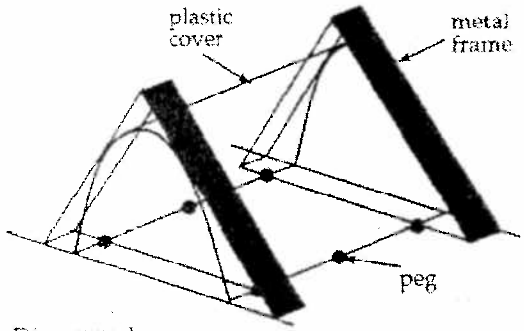


Diagram 1

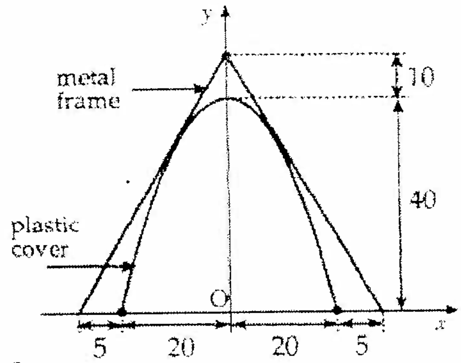


Diagram 2

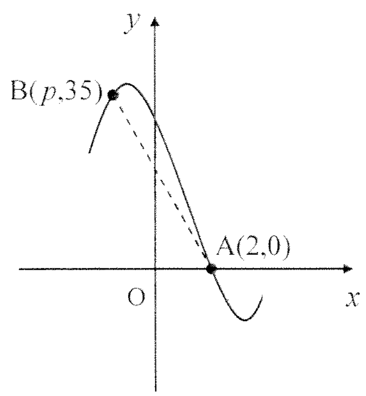
(*a*) Show that the equation of the parabolic end is $y = 40 - \dfrac{x^2}{10}$, $-20 \le x \le 20$. **(4)**

(*b*) Show that the triangular frame touches the cover without disturbing the parabolic shape. **(7)**

7. A curve has as its equation $y = x^3 - kx^2 - 16x + 32$.

Part of the graph of this curve is shown below.

The diagram is not drawn to scale.



(a) If the curve crosses the x-axis at A(2,0), find k. **3**

(b) The point B(p,35) also lies on this curve, find the value of p, *where* $p \in \mathbf{Z}$ **3**

(c) Calculate the size of the angle between the line AB and the x-axis in the positive direction. **Give your answer to the nearest degree.** **2**

8. Given that $\int_0^a (2x)\, dx$ is equal to the **derivative** of $3a^2 - 9a$, find a. **4**

[END OF QUESTION PAPER]