	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	ans: A(3, 9) **(4 marks)** •1 equates 2 equations and collects to LHS •2 factorises •3 solves for x chooses appropriate value •4 substitutes and states point A	•1 $4x^2 - x^3 = 3x; 4x^2 - x^3 - 3x = 0$ •2 $x(x-3)(x-1) = 0$ •3 $x = 0, 1, 3; x = 3$ •4 $y = 3 \times 3 = 9$; A(3, 9)
(b)	ans: $y + 3x = 18$ **(4 marks)** •1 know to take derivative •2 knows to substitute •3 evaluates to find gradient •4 substitutes into $y - b = m(x - a)$	•1 $\frac{dy}{dx} = 8x - 3x^2$ •2 $8(3) - 3(3)^2$ •3 $m = -3$ •4 $y - 9 = -3(x - 3)$
22(a)	ans: proof **(3 marks)** •1 uses right triangle and finds third side •2 writes ratio of sin •3 shows simplification to answer OR •1 knows to sub in appropriate formula •2 subs in formula and starts to simplify •3 shows simplification to answer	•1 triangle with sides $\sqrt{2}$, 10 and $\sqrt{98}$ •2 $\sin x = \frac{\sqrt{98}}{10}$ •3 $\sqrt{98} = \sqrt{49} \times \sqrt{2} = 7\sqrt{2}$ •1 $\sin^2 x = 1 - \cos^2 x$ •2 $\sin^2 x = 1 - \left(\frac{\sqrt{2}}{10}\right)^2 = 1 - \frac{2}{10} = \frac{98}{100}$ •3 $\sin x = \sqrt{\frac{98}{100}} = \frac{\sqrt{98}}{10} = \frac{7\sqrt{2}}{10}$
(b)	ans: proof **(4 marks)** •1 knows to expand •2 correct expansion •3 subs values •4 simplifies to answer	•1 evidence of expansion •2 $\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$ •3 $\frac{7\sqrt{2}}{10} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{2}}{10} \times \frac{1}{\sqrt{2}}$ •4 $\frac{7\sqrt{2}}{10\sqrt{2}} + \frac{\sqrt{2}}{10\sqrt{2}} = \frac{8\sqrt{2}}{10\sqrt{2}} = \frac{8}{10} = 0 \cdot 8$

	Give 1 mark for each •	Illustration(s) for awarding each mark
23(a)	**ans:** proof **(2 marks)** •1 cross multiplies •2 multiplies brackets and collects terms	•1 $5x(x+k^2)=4(x^2-k^2)$ •2 $5x^2+5k^2x=4x^2-4k^2$ $5x^2+5k^2x-4x^2+4k^2=0$
(b)	**ans:** $k=\pm\frac{4}{5}$ **(4 marks)** •1 knows condition for equal roots •2 substitutes values •3 factorises •4 solves and chooses values for k	•1 $b^2-4ac=0$ [stated or implied] •2 $(5k^2)^2-4\times1\times4k^2=0$ $25k^4-16k^2=0$ •3 $k^2(5k-4)(5k+4)=0$ •4 $\pm\frac{4}{5}$, as $k\neq0$
24(a)	**ans:** proof **(4 marks)** •1 finds expressions for missing dimensions •2 finds area of triangles •3 subtracts from area of rectangle •4 simplifies to answer	•1 $(6-2x)$ and $(4-x)$ •2 $x(6-2x)$ and $2x(4-x)$ •3 $24-(6x-2x^2+8x-2x^2)$ •4 $24-6x+2x^2-8x+2x^2$
(b)	**ans:** $\frac{7}{4}$; $11\frac{3}{4}$ **(5 marks)** •1 knows to make derivative equal to 0 •2 finds derivative a •3 solves for x and justifies •4 subs value to find area •5 answer	•1 $\frac{dy}{dx}=0$ •2 $\frac{dy}{dx}=8x-14=0$ •3 $x=\frac{7}{4}$; table of values or second derivative •4 $a=4\left(\frac{7}{4}\right)^2-14\left(\frac{7}{4}\right)+24$ •5 $11\frac{3}{4}$

Total: 70 marks

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	ans: $a = 1;\ b = 4,\ c = -29$ **(4 marks)** •1 finds gradient of BD •2 finds gradient of AC •3 subs into $y - b = m(x - a)$ and rearranges •4 states values of a, b and c	•1 $m_{BD} = 4$ [from equation] •2 $m_{AC} = -\frac{1}{4}$ •3 $y - 8 = -\frac{1}{4}(x+3);\ x + 4y - 29 = 0$ •4 $a = 1;\ b = 4,\ c = -29$
(b)	ans: E(5, 6) **(3 marks)** •1 knows to use system of equations •2 solves for x and y •3 states coordinates of E	•1 evidence of equating one variable •2 $x = 5;\ y = 6$ •3 E(5, 6)
(c)	ans: C(13, 4) **(2 marks)** •1 appropriate method •2 answer	•1 evidence of 'stepping out' or other method •2 C(13, 4)
2(a)	ans: proof **(3 marks)** •1 knows to substitute •2 substitutes correctly •3 clearly simplifies to answer	•1 evidence of sub. one function in other •2 $f\left(\frac{1}{x-1}\right) = \frac{4}{x-1} + 1$ •3 $\frac{4 + x - 1}{x - 1} = \frac{x+3}{x-1}$
(b)	ans: $p = 2$ **(4 marks)** •1 substitute for x •2 knows to multiply by conjugate surd •3 multiplies and simplifies •4 states value of p	•1 $\frac{\sqrt{5}+3}{\sqrt{5}-1}$ •2 $\frac{\sqrt{5}+3}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$ •3 $\frac{5 + 4\sqrt{5} + 3}{4} = \frac{8 + 4\sqrt{5}}{4} = 2 + \sqrt{5}$ •4 $p = 2$

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	**ans:** P(1, 0); Q(– 2, 27) **(5 marks)**	
	•[1] knows derivative = 0 at S.P. •[2] takes derivative and factorises •[3] solves for x and chooses appropriate value •[4] substitutes to find y – coordinate •[5] states coordinates of P and Q	•[1] $f'(x) = 0$ at SP [stated or implied] •[2] $6x^2 + 6x - 12 = 0$; $6(x+2)(x-1) = 0$ •[3] $x = -2$ or 1 •[4] $f(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) + 7 = 27$ •[5] P(1, 0); Q(– 2, 27)
(b)	**ans:** 40·5 units^2 **(4 marks)**	
	•[1] sets up integral •[2] integrates expression •[3] substitutes values •[4] evaluates	•[1] $\int_{-2}^{1} 2x^3 + 3x^2 - 12x + 7\ dx$ •[2] $\left[\frac{x^4}{2} + x^3 - 6x^2 + 7x\right]_{-2}^{1}$ •[3] $\left(\frac{(1)^4}{2} + (1)^3 - 6(1)^2 + 7(1)\right) - \left(\frac{(-2)^4}{2} + (-2)^3 - 6(-2)^2 + 7(-2)\right)$ •[4] 40·5 units^2
4	**ans:** 19·5°, 90°, 160·5°, 270° **(5 marks)**	
	•[1] subs for sin $2x°$ •[2] multiplies and simplifies •[3] factorises •[4] finds two solutions •[5] finds further two solutions	•[1] $15(2\sin x° \cos x°)$..... •[2] $30\sin x° \cos x° - 10\cos x° = 0$ •[3] $10\cos x°(3\sin x° - 1) = 0$ •[4] $\sin x° = \frac{1}{3}$; $x = 19\cdot5°, 160\cdot5°$ •[5] $\cos x° = 0$; $x = 90°, 270°$

	Give 1 mark for each •	Illustration(s) for awarding each mark
5(a)	ans: proof (2 marks) •1 substitutes U_0 and finds U_1 •2 substitutes U_1 and finds U_2	•1 $U_1 = \frac{a}{4} \times 16 + 12 = 4a + 12$ •2 $U_2 = \frac{a}{4}(4a + 12) + 12 = a^2 + 3a + 12$
(b)	ans: $a = 3$ (3 marks) •1 equates U_2 to 30 •2 collects terms to LHS and factorises •3 solves for x and discards	•1 $a^2 + 3a + 12 = 30$ •2 $a^2 + 3a - 18 = 0; (a + 6)(a - 3) = 0$ •3 $a = -6, 3; a = 3$
(c)	ans: 48 (3 marks) •1 knows condition for limit •2 knows how to find limit •3 answer	•1 limit exists since $-1 < \frac{3}{4} < 1$ •2 $L = \frac{12}{1 - 0 \cdot 75} = \frac{12}{0 \cdot 25}$ •3 48
6(a)	(a) •1 $y = ax^2 + bx + c$ •2 $(0, 40) \Rightarrow c = 40$ •3 symmetry $\Rightarrow b = 0$ •4 $(20, 0) \Rightarrow a = -\frac{1}{10}$	
(b)	(b) •5 strategy: find equ of line and solve with parabola •6 e.g. gradient of left line = 2 •7 $y = 2x + 50$ •8 $2x + 50 = 40 - \frac{1}{10}x^2$ •9 $x^2 + 20x + 100 = 0$ •10 $b^2 - 4ac = 0$ or $(x - 10)^2 = 0$ •11 equal roots so line is tangent to parabola	

	Give 1 mark for each •	Illustration(s) for awarding each mark
7(a)	**ans:** $k = 2$ **(3 marks)** •1 knows to use synthetic division •2 makes remainder = 0 •3 solves for k	•1 evidence •2 $8 - 4k = 0$ •3 $k = 2$
(b)	**ans:** $p = -3$ **(3 marks)** •1 equates function to 35 •2 collect terms to LHS and equates to 0 •3 uses synthetic division to find root	•1 $p^3 - 2p^2 - 16p + 32 = 35$ •2 $p^3 - 2p^2 - 16p - 3 = 0$ •3 $p = -3$
(c)	**ans:** 98° **(2 marks)** •1 finds gradient of AB •2 takes $\tan^{-1}$ and states angle	•1 $m_{AB} = \dfrac{35 - 0}{-3 - 2} = -7$ •2 $\tan^{-1}(7) = 82^\circ$: angle = 98°
8	**ans:** $a = 3$ **(4 marks)** •1 evaluates integral •2 finds derivative •3 makes integral = derivative •4 factorises and solves	•1 $\left[x^2\right]_0^a = a^2$ •2 $\dfrac{d}{da} = 6a - 9$ •3 $a^2 = 6a - 9$; $a^2 - 6a + 9 = 0$ •4 $(a - 3)(a - 3) = 0$; $a = 3$

Total: ~~60~~ 61 marks

Higher Grade - Paper 1 2009/2010

ANSWERS - Section A

1	B
2	D
3	D
4	C
5	C
6	A
7	B
8	C
9	B
10	C
11	D
12	A
13	B
14	B
15	C
16	D
17	A
18	A
19	C
20	B

	A	B	C	D
1	▭	■	▭	▭
2	▭	▭	▭	■
3	▭	▭	▭	■
4	▭	▭	■	▭
5	▭	▭	■	▭
6	■	▭	▭	▭
7	▭	■	▭	▭
8	▭	▭	■	▭
9	▭	■	▭	▭
10	▭	▭	■	▭
11	▭	▭	▭	■
12	■	▭	▭	▭
13	▭	■	▭	▭
14	▭	■	▭	▭
15	▭	▭	■	▭
16	▭	▭	▭	■
17	■	▭	▭	▭
18	■	▭	▭	▭
19	▭	▭	■	▭
20	▭	■	▭	▭