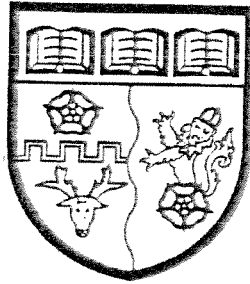


Established 1791



## *Fortrose Academy*

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Mathematics  
Higher Prelim Examination 2008/2009  
Paper 1  
Assessing Units 1 & 2

NATIONAL  
QUALIFICATIONS

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.



### Read carefully

- 1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2008/2009 (Section A)**.
- 2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
- 3 Make sure you write your **name, class** and **teacher** on the answer sheet provided.
- 4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
- 5 There is **only one correct** answer to each question.
- 6 Rough working should **not** be done on your answer sheet.
- 7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

### Sample Question

A line has equation  $y = 4x - 1$ .

If the point  $(k, 7)$  lies on this line, the value of  $k$  is

- A 2
- B 27
- C 1.5
- D -2

The correct answer is  $A \rightarrow 2$ . The answer A should then be clearly marked in pencil with a horizontal line (see below).



### Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to D.



## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

SECTION A

ALL questions should be attempted

1. If  $f(x) = 2x^{\frac{3}{2}}$  then  $f'(4)$  equals
- A 16  
B 4  
C  $25\frac{3}{5}$   
D 6
2. Triangle ABC has vertices A  $(-3, -3)$ , B  $(12, -1)$  and C  $(6, 11)$ .  
The gradient of the **altitude** through B is
- A  $-\frac{9}{14}$   
B  $\frac{14}{9}$   
C  $-\frac{3}{8}$   
D  $\frac{8}{3}$
3. The remainder when  $x^3 - 11x + 10$  is divided by  $(x + 3)$  is
- A 52  
B 16  
C 4  
D -24
4. The point P  $(8, y)$  lies on the circle with equation  $x^2 + y^2 - 12x + 4y + 20 = 0$ .  
The value(s) of  $y$  is/are
- A 2 only  
B -6 only  
C -6 and 2  
D 6 and -2

5. A sequence is defined by the recurrence relation  $U_{n+1} = aU_n - 5$  with  $U_0 = 10$ .

An expression in terms of  $a$  for  $U_2$  is

- A  $10a - 5$
- B  $10a^2 - 5$
- C  $10a^2 - 5a - 5$
- D  $10a^2$

6.  $\int_0^1 4x(x^2 - 2) \, dx$  is

- A  $-3$
- B  $-4$
- C  $0$
- D  $12$

7. The equation  $2x^2 + 8 = kx$  has **no real roots**.  $k$  must take the values

- A  $\pm 8$
- B  $-8 < k < 8$
- C  $k < -8$  or  $k > 8$
- D undefined

8. For which value(s) of  $x$  is the function  $f(x) = \frac{3}{(x+3)(x-2)}$  undefined?

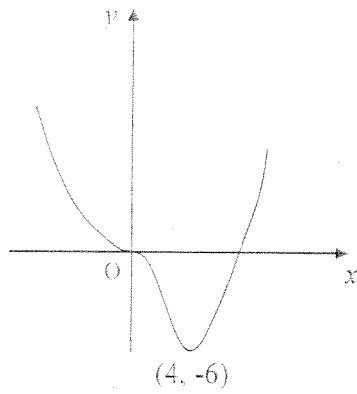
- A  $3$
- B  $3$  and  $-2$
- C  $-3$  and  $2$
- D  $-6$

9. The line  $ax - 3y + 6 = 0$  is perpendicular to the line with equation  $4x + y - 3 = 0$ .

The value of  $a$  is

- A  $12$
- B  $\frac{3}{4}$
- C  $-4$
- D  $\frac{1}{4}$

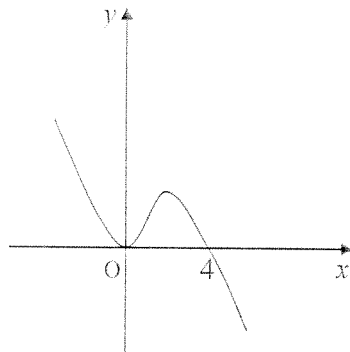
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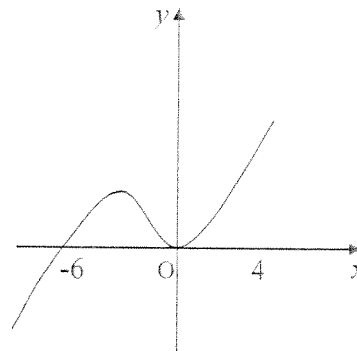
The diagram shows part of the graph of  $y = f(x)$ .  
It has stationary points at  $(0, 0)$  and  $(4, -6)$ .

Which of the following could be part of the graph of the derived function  $y = f'(x)$ ?

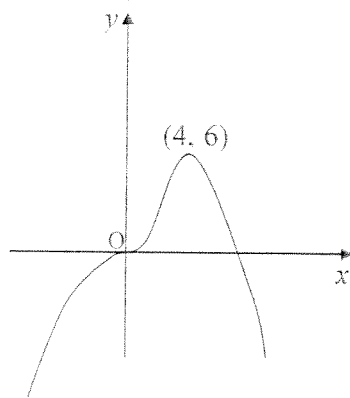
A



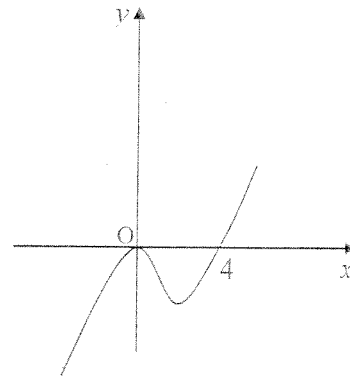
B



C



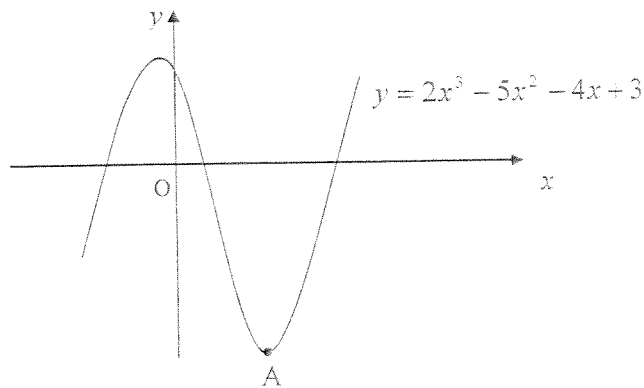
D



11. The two sequences defined by the recurrence relations  $U_{n+1} = 0.5U_n + 20$  and  $V_{n+1} = 0.2V_n + k$  have the same limit. The value of  $k$  is

- A 8
- B 20
- C 40
- D 32

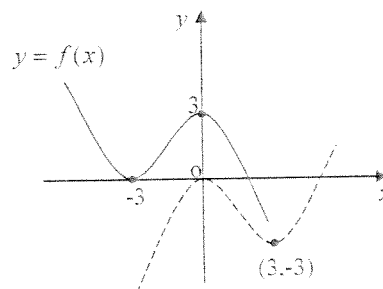
12. The diagram shows part of the curve with equation  $y = 2x^3 - 5x^2 - 4x + 3$ .



The  $x$ -coordinate of the point  $A$  is

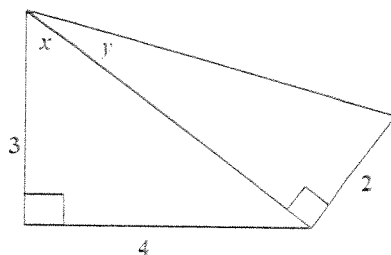
- A  $\frac{1}{3}$   
 B 2  
 C 3  
 D -9
13. The function  $f$  is defined as  $f(x) = \frac{x-6}{x}$ ,  $x \neq 0$ . The value of  $f(f(3))$  equals
- A 7  
 B -7  
 C -5  
 D -1
14. The diagram shows the graph of  $y = f(x)$  as a full line and the graph of a related function as a broken line. The equation of the related function is

- A  $y = -f(x) - 3$   
 B  $y = f(x-6) - 3$   
 C  $y = f(-x) - 3$   
 D  $y = f'(x)$





15. The diagram shows two right-angled triangles with lengths as shown.



The exact value of  $\cos(x + y)$  is

- A  $\frac{8}{\sqrt{29}}$
- B  $\frac{7}{5\sqrt{29}}$
- C  $\frac{23}{5\sqrt{29}}$
- D  $\frac{3}{\sqrt{29}}$
16. A circle has centre  $A(1, 3)$  and radius  $\sqrt{5}$ . Another circle has centre  $B(9, 7)$  and radius  $3\sqrt{5}$ . Which of the following is true for these two circles?
- A they intersect at two points
- B they touch externally
- C they touch internally
- D they do not intersect or touch
17. The maximum value of  $\frac{12}{x^2 - 4x + 10}$  is
- A 2
- B -2
- C 6
- D -6

18. A ball is thrown upwards reaching a height of ' $h$ ' metres after ' $t$ ' seconds where  $h(t) = 2 + 12t - 3t^2$ . The time taken, in seconds, to reach its maximum height is

A 2  
B 3  
C 4  
D 5

19. The exact value of  $\sin \frac{2\pi}{3} - \cos \frac{7\pi}{6}$  is

A 0  
B 1  
C  $\sqrt{3}$   
D  $\frac{\sqrt{3}}{4}$

20.  $(x, y)$  is a solution for the system of equations

$$x^2 + 7y^2 = 16$$

$$x - 3y = 0.$$

Possible values for  $x + y$  are

(1) 0    (2) 4    (3) -4

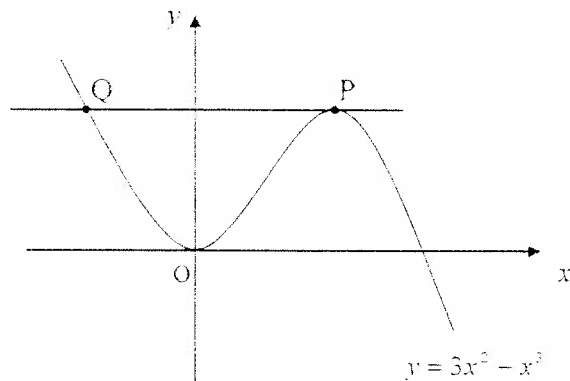
A (2) and (3) only  
B (2) only  
C (1) only  
D (1), (2) and (3)

[ END OF SECTION A ]

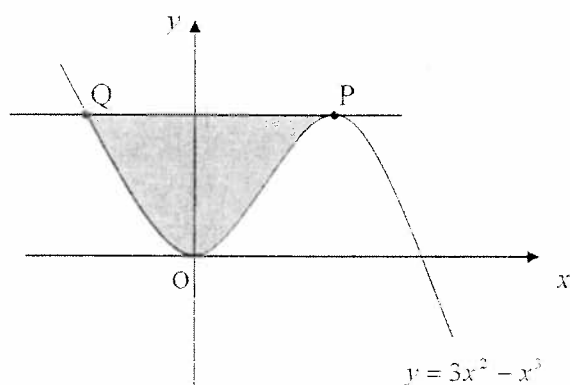
## SECTION B

ALL questions should be attempted

21. Part of the graph of the curve with equation  $y = 3x^2 - x^3$  is shown below. The diagram is not drawn to scale.



- (a) Establish the coordinates of the stationary point P. 4
- (b) The horizontal line through P meets the curve again at Q. Find the coordinates of Q. 3
- (c) Hence calculate the shaded area shown in the diagram below.

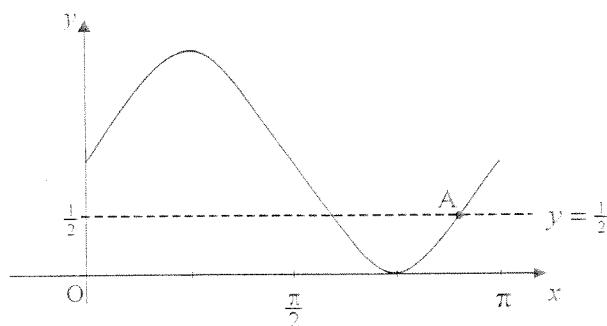


6

22. Two functions, defined on suitable domains, are given as  $f(x) = x^2 - 1$  and  $g(x) = 2 - x$ .

- (a) Show that  $f(g(a))$  can be expressed in the form  $pa^2 + qa + r$  and write down the values of  $p$ ,  $q$  and  $r$ . 4
- (b) Hence find  $a$  if  $f(g(a)) = 8$  and  $a > 0$ . 2

23. The diagram below shows part of the graph of  $y = \sin 2x + 1$ , for  $0 \leq x \leq \pi$ , and the line with equation  $y = \frac{1}{2}$ .



Find the coordinates of the point A.

4

24. A recurrence relation is defined by the formula  $U_{n+1} = 0.6U_n + 24$ .

- Establish the limit of this sequence. 2
- Given now that  $U_1$  is exactly half of this limit, find  $U_0$ , the initial value of the sequence. 2
- A second recurrence relation in the form  $U_{n+1} = aU_n + b$  has the **same limit** as the sequence above and is such that  $b = 90a$ .

Find the values of  $a$  and  $b$  in this second sequence.

3

[ END OF SECTION B ]

[ END OF QUESTION PAPER ]