

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	ans: $2y - x = -2$ (3 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> finds midpoint of BC</li> <li>•<sup>2</sup> establishes gradient of AM</li> <li>•<sup>3</sup> substitutes in general equation</li> </ul>
(b)	ans: D(4,1) (3 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> realising <math>y = 1</math></li> <li>•<sup>2</sup> substitutes into equation</li> <li>•<sup>3</sup> states coordinates of D</li> </ul> <p> <math>\bullet^1 y = 1</math>  <math>\bullet^2 2(1) - x = -2; x = 4</math>  <math>\bullet^3 D(4,1)</math> </p>
(c)	ans: proof (3 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> finds gradient of BD</li> <li>•<sup>2</sup> knows condition for perp. lines</li> <li>•<sup>3</sup> makes statement re perpendicular</li> </ul> <p> <math>\bullet^1 m_{BD} = -2</math>  <math>\bullet^2 m_1 \times m_2 = -1</math> [stated or implied]  <math>\bullet^3 \frac{1}{2} \times -2 = -1</math> so AM and BD are perp.     </p>
2(a)	ans: P( $1, -\frac{25}{2}$ ) (4 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows to take derivative and equate to 0</li> <li>•<sup>2</sup> takes derivative</li> <li>•<sup>3</sup> solves to find x - coordinate</li> <li>•<sup>4</sup> substitutes to find y - coordinate</li> </ul> <p> <math>\bullet^1 \frac{dy}{dx} = 0</math>  <math>\bullet^2 3x^2 - 15x + 12 = 0</math>  <math>\bullet^3 x = 1</math> [or 4]  <math>\bullet^4 y = 1^3 - \frac{15}{2}(1) + 12(1) - 18 = -\frac{25}{2}</math> </p>
(b)	ans: Q(6,0) (3 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows to make <math>y = 0</math></li> <li>•<sup>2</sup> uses synthetic division to find x</li> <li>•<sup>3</sup> states coordinates of Q</li> </ul> <p> <math>\bullet^1 y = 0</math>  <math>\bullet^2 \begin{array}{r rrrr} 6 &amp; 1 &amp; -\frac{15}{2} &amp; 12 &amp; -18 \\ &amp; &amp; 6 &amp; -9 &amp; 18 \\ \hline &amp; 1 &amp; -\frac{3}{2} &amp; 3 &amp; 0 \end{array}</math>  <math>\bullet^3 Q(6,0)</math> </p>

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3(a)	ans: 32.6 gigatonnes (3 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> correct multiplier</li> <li>•<sup>2</sup> completes calculation</li> <li>•<sup>3</sup> calculation and correct rounding</li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> 0.96</li> <li>•<sup>2</sup> <math>0.96^5 \times 40</math></li> <li>•<sup>3</sup> 32.6 gigatonnes</li> </ul>
(b)	ans: 31 gigatonnes (3 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> sets up recurrence relation</li> <li>•<sup>2</sup> knows to calculate 3 figures</li> <li>•<sup>3</sup> final answer</li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>U_{n+1} = 0.96^5 U_n + 3.8</math></li> <li>•<sup>2</sup> 1<sup>st</sup> year: 36.4; 2<sup>nd</sup> year: 33.4795</li> <li>•<sup>3</sup> 3<sup>rd</sup> year: 31 gigatonnes</li> </ul>
(c)	ans: upper 20.6; lower 16.8 (3 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows limit exists</li> <li>•<sup>2</sup> finds upper limit</li> <li>•<sup>3</sup> finds lower limit</li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> limit exists since <math>-1 &lt; 0.96^5 &lt; 1</math></li> <li>•<sup>2</sup> <math>L = \frac{3.8}{1 - (0.96)^5} = 20.6</math></li> <li>•<sup>3</sup> <math>20.6 - 3.8 = 16.8</math></li> </ul>
4(a)	ans: $a = -2$ (2 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> finds expression for <math>f(g(-2))</math></li> <li>•<sup>2</sup> equates to -1 and solves for <math>a</math></li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(g(-2)) = f(-1) = 1 + a</math></li> <li>•<sup>2</sup> <math>a = -2</math></li> </ul>
(b)	ans: $x = -2, 0, 2$ (5 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> substitutes</li> <li>•<sup>2</sup> simplifies</li> <li>•<sup>3</sup> equates to 2</li> <li>•<sup>4</sup> factorises</li> <li>•<sup>5</sup> solves for <math>x</math></li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(f(x)) = (x^2 - 2)^2 - 2</math></li> <li>•<sup>2</sup> <math>x^4 - 4x^2 + 2</math></li> <li>•<sup>3</sup> <math>x^4 - 4x^2 + 2 = 2: x^4 - 4x^2 = 0</math></li> <li>•<sup>4</sup> <math>x^2(x^2 - 4) = 0</math></li> <li>•<sup>5</sup> <math>x = -2, 0, 2</math></li> </ul>
5(a)	ans: $x = 1$ (2 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> realises <math>y = 0</math>; equates to 0</li> <li>•<sup>2</sup> solves for <math>x</math></li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>3x^2 - 6x + 3 = 0</math></li> <li>•<sup>2</sup> <math>3(x - 1)^2 = 0; x = 1</math></li> </ul>
(b)	ans: $b = 2$ (5 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> integrates expression</li> <li>•<sup>2</sup> substitutes values</li> <li>•<sup>3</sup> simplifies, equates to 1, rearranges</li> <li>•<sup>4</sup> uses synthetic division to solve</li> <li>•<sup>5</sup> realises one solution; discards <math>b^2 - b + 1</math></li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>[x^3 - 3x^2 + 3x]^b</math></li> <li>•<sup>2</sup> <math>(b^3 - 3b^2 + 3b) - (1 - 3 + 3)</math></li> <li>•<sup>3</sup> <math>b^3 - 3b^2 + 3b - 2 = 0</math></li> <li>•<sup>4</sup> <math display="block">\begin{array}{r rrrr} 2 &amp; 1 &amp; -3 &amp; 3 &amp; -2 \\ &amp; &amp; 2 &amp; -2 &amp; 2 \\ \hline &amp; 1 &amp; -1 &amp; 1 &amp; 0 \end{array}</math></li> <li>•<sup>5</sup> <math>b = 2</math></li> </ul>

	Give 1 mark for each •	Illustration(s) for awarding each mark
5(c)	ans: $y = 6x - 9$ (4 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> finds 'c'</li> <li>•<sup>2</sup> knows to differentiate</li> <li>•<sup>3</sup> substitutes to find gradient</li> <li>•<sup>4</sup> substitutes into general equation</li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> when <math>x = 2, y = 3</math>; Q(2,3)</li> <li>•<sup>2</sup> <math>\frac{dy}{dx} = 6x - 6</math></li> <li>•<sup>3</sup> when <math>x = 2</math>; <math>\frac{dy}{dx} = 6</math></li> <li>•<sup>4</sup> <math>y - 3 = 6(x - 2)</math></li> </ul>
6(a)	ans: (-2,1) (3 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> substitutes eq.of line in eq. of circle</li> <li>•<sup>2</sup> simplifies and solves for <math>x</math></li> <li>•<sup>3</sup> substitutes to find <math>y</math></li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>x^2 + (2x + 5)^2 - 4x + 2(2x + 5) - 15 = 0</math></li> <li>•<sup>2</sup> <math>5(x + 2)^2 = 0; x = -2</math></li> <li>•<sup>3</sup> <math>y = 2(-2) + 5; y = 1</math></li> </ul>
(b)	ans: $(x + 6)^2 + (y - 3)^2 = 20$ (3 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> establishes coordinates of B</li> <li>•<sup>2</sup> finds <math>r^2</math></li> <li>•<sup>3</sup> substitutes into general circle equation</li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> B(-6,3)</li> <li>•<sup>2</sup> <math>r^2 = 20</math></li> <li>•<sup>3</sup> <math>(x + 6)^2 + (y - 3)^2 = 20</math></li> </ul>
7(a)	ans: $AE = \sqrt{5}; BC = 4$ (2 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows to use Pythagoras'</li> <li>•<sup>2</sup> finds two lengths</li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> evidence</li> <li>•<sup>2</sup> <math>AE = \sqrt{5}; BC = \sqrt{(25 - 9)} = 4</math></li> </ul>
(b)	ans: proof (4 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows to expand</li> <li>•<sup>2</sup> finds expression for ratios</li> <li>•<sup>3</sup> subs and starts to simplify</li> <li>•<sup>4</sup> completes simplification to answer</li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\cos(x + y) = \cos x \cos y - \sin x \sin y</math></li> <li>•<sup>2</sup> <math>\cos x = \frac{4}{5}; \cos y = \frac{2}{\sqrt{5}}; \sin x = \frac{3}{5}; \sin y = \frac{1}{\sqrt{5}}</math></li> <li>•<sup>3</sup> <math>(\frac{4}{5} \times \frac{2}{\sqrt{5}}) - (\frac{3}{5} \times \frac{1}{\sqrt{5}}); \frac{8}{5\sqrt{5}} - \frac{3}{5\sqrt{5}}</math></li> <li>•<sup>4</sup> <math>\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}</math></li> </ul>
8(a)	ans: $m = 2400\text{ml}$ (4 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows to differentiate and equate to 0</li> <li>•<sup>2</sup> differentiates</li> <li>•<sup>3</sup> solves for <math>x</math></li> <li>•<sup>4</sup> justifies maximum</li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>H'(m) = 0</math></li> <li>•<sup>2</sup> <math>4 - \frac{1}{600}m = 0</math></li> <li>•<sup>3</sup> <math>m = 2400</math></li> <li>•<sup>4</sup> table of values; second derivative</li> </ul>
(b)	ans: 4800 feet (1 mark)	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows to sub into function and evaluate</li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>4(2400) - \frac{(2400)^2}{1200} = 4800\text{feet}</math></li> </ul>

Total: 60 marks

