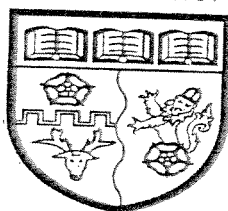


Fortrose Academy

Established 1791



Prelim Examination 2006 / 2007
(Assessing Units 1 & 2)

MATHEMATICS

Higher Grade - Paper II

Time allowed - 1 hour 30 minutes

Read Carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

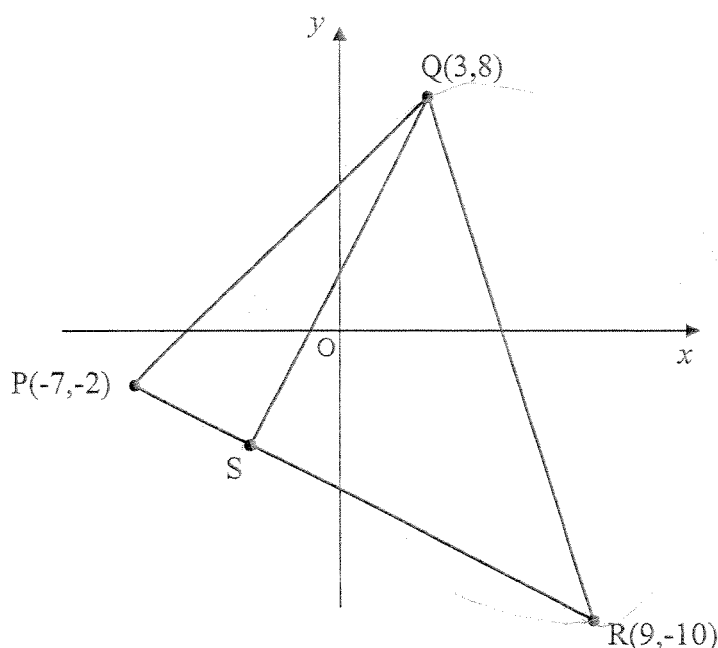
$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

All questions should be attempted

1. Triangle PQR has as its vertices $P(-7,-2)$, $Q(3,8)$ and $R(9,-10)$ as shown.



- | | | |
|-----|--|---|
| (a) | Find the equation of side PR. | 2 |
| (b) | Find the equation of the altitude QS. | 3 |
| (c) | Hence find the coordinates of S, the point where the altitude QS meets side PR. | 4 |
| (d) | Establish the equation of the circle which passes through the points Q, S and R. | 4 |

2. A recurrence relation is defined as $u_{n+1} = 0.75u_n + 12$.

Given that $U_0 = 32$, find the **difference** between the limit of the sequence and the third term, U_3 .

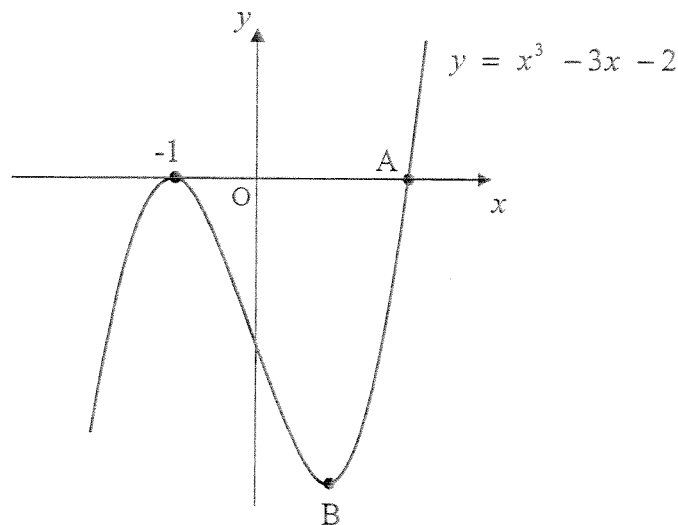
5

3. A curve has as its equation $y = (x-6)^2 + 8$.

Given that the line with equation $y = 2x - 5$ is a tangent to this curve, establish the coordinates of the point T, the point of contact between the curve and the line.

4

4. Part of the graph of the curve with equation $y = x^3 - 3x - 2$ is shown below. The curve passes through the point $(-1, 0)$.



Find, algebraically, the coordinates of the points A and B.

7

5. A circle has as its equation $(x - 9)^2 + (y + 1)^2 = 117$.

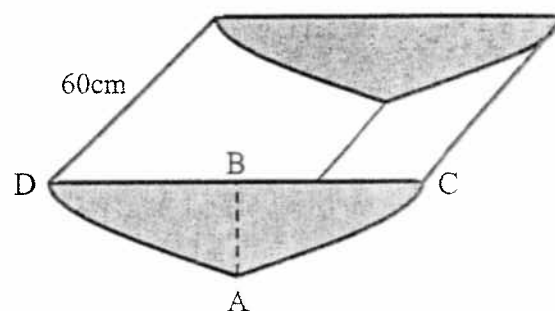
- (a) Given that the point $A(3, k)$ lies on this circle, find k where $k > 0$. 4
- (b) Find the equation of the tangent to this circle at the point A. 4
- (c) Show clearly that this tangent passes through the centre of the circle with equation $x^2 + y^2 + 6x - 8y + 12 = 0$. 2

6. The functions f and g , defined on suitable domains, are given as

$$f(x) = \frac{x^2}{2} - \frac{3}{4} \quad \text{and} \quad g(x) = \frac{5ax}{4} - a, \quad \text{where } a \text{ is a constant.}$$

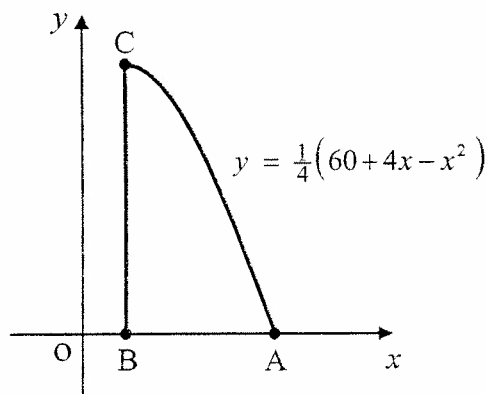
- (a) Given that $f(a) = g(1)$, find the value of a , where $a < 0$. 4
- (b) With a taking this value, find the **rate of change** of g . 2

7. A small feeding trough is shown opposite.
- The end face has an axis of symmetry AB.
- Edge CD is perpendicular to the axis of symmetry.



When the end face is rotated through 90° and then halved along the axis of symmetry, shape ABC can be placed on a coordinate diagram as shown below.

AB lies along the x -axis with the curved edge CA being part of the curve with equation $y = \frac{1}{4}(60 + 4x - x^2)$.



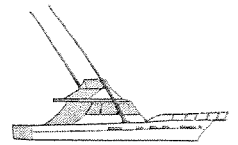
- Establish the coordinates of A and B. 4
- Hence calculate the **area** of shape ABC given that all the units are in centimetres. 4
- Given that the trough is a prism and measures 60cm from back to front, calculate the volume of feed the trough can hold when full, giving your answer correct to the nearest litre. 3

8. Solve algebraically the equation

$$2 \sin x^\circ - 3 = 5 \cos 2x^\circ, \text{ where } 0 < x < 360.$$

6

9. The captain of a small pleasure boat wishes to take a group of passengers from one island to the next, a journey of 100 kilometres.



The amount of fuel used is dependent upon the speed, v kilometres per hour, of the boat.

- (a) Given that the rate of fuel used is $(1 + 0.0000625v^3)$ gallons per hour, **show clearly** that the total fuel used, F , for this 100 kilometre journey is given by

$$F = \frac{100}{v} + 0.00625v^2 \text{ gallons.} \quad 3$$

- (b) Hence find the speed which keeps the amount of fuel used to a minimum and the amount of fuel needed, at this speed, for the voyage. 5

[END OF QUESTION PAPER]

