Higher Paper 1

Prelim 2006/2007

Total Marks 60

① (a) $h(x) = f(g(x))$

$= f(x-1)$

$= (x-1)((x-1)^2-1)$

$= (x-1)(x^2-2x)$

$= x^3-2x^2-x^2+2x$

$= x^3-3x^2+2x$

$\underline{a=1 \quad b=-3 \quad c=2}$

4.

(b) $h(x) = 6$

$x^3-3x^2+2x = 6$

$x^3-3x^2+2x-6 = 0$

✓ method.

3	1	-3	2	-6
		3	0	6
	1	0	2	0

so $x-3$ is a factor

$x^3-3x^2+2x-6 = (x-3)(x^2+2)$

$(x-3)(x^2+2) = 0$

$\Rightarrow \underline{x=3}$ or $x^2=-2$

not possible

Hence $x=3$ is the only possible solution.

4.

② (a) $4y = x+13$

$y = \frac{1}{4}x + \frac{13}{4}$.

$m_{L_1} = \frac{1}{4} = m_{L_2}$

equation L_2 $\quad y-b = m(x-a) \quad m = \frac{1}{4} \quad (a,b) = (0,-1)$

$y+1 = \frac{1}{4}(x-0)$

$4y+4 = x$

$4y = x-4$

cuts x-axis $\quad y=0 \Rightarrow x=4 \quad \underline{\underline{A(4,0)}}$ ✓ (3)

(b) $m_{AB} = -4$ ✓

Equation of AB $\quad y-b = m(x-a) \qquad m=-4 \quad (a,b)=(4,0)$

$y-0 = -4(x-4)$

$\underline{\underline{y+4x=16}}$ ✓

Point of intersection

Solve $\quad y+4x=16 \quad \ldots (1)$ ✓

$4y-x=13 \quad \ldots (2)$

$(1)+4(2) \qquad 17y=68$

$y=4$ ✓ $\Rightarrow x=3$ ✓ (5)

(c) $A(4,0) \quad B(3,4)$

$AB = \sqrt{(4-0)^2+(3-4)^2}$ ✓

$= \sqrt{16+1}$

$\underline{\underline{AB = \sqrt{17}}}$ ✓ (2)

(3) $(p^2+11)x^2 - 12px + p^2 = 0$

$a = p^2+11 \qquad b=-12p \qquad c=p^2$ ✓

For equal roots

$b^2-4ac=0$ ✓

$\Rightarrow (-12p)^2 - 4(p^2+11)p^2 = 0$ ✓

$144p^2 - 4p^4 - 44p^2 = 0$

$100p^2 - 4p^4 = 0$ ✓

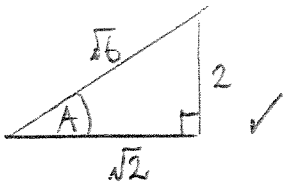
$4p^2(25-p^2)=0$

$4p^2(5-p)(5+p)=0$

$p=0 \quad p=5 \quad$ or $\quad p=-5$ ✓

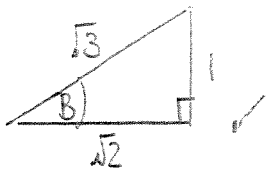
Since $p>0$, $\underline{\underline{p=5}}$ ✓ (6)

(4) $\sin A = \frac{2}{\sqrt{6}}$



✓

~~Sit~~ $\cos B = \frac{\sqrt{2}}{\sqrt{3}}$



✓

$$\cos(A-B) = \cos A \cos B + \sin A \sin B \quad ✓$$

$$= \frac{\sqrt{2}}{\sqrt{6}} \cdot \frac{\sqrt{2}}{\sqrt{3}} + \frac{2}{\sqrt{6}} \cdot \frac{1}{\sqrt{3}} \quad ✓$$

$$= \frac{2}{\sqrt{18}} + \frac{2}{\sqrt{18}}$$

$$= \frac{4}{\sqrt{18}}$$

So $3\cos(A-B) = \frac{12}{\sqrt{18}}$

$$= \frac{12}{3\sqrt{2}}$$

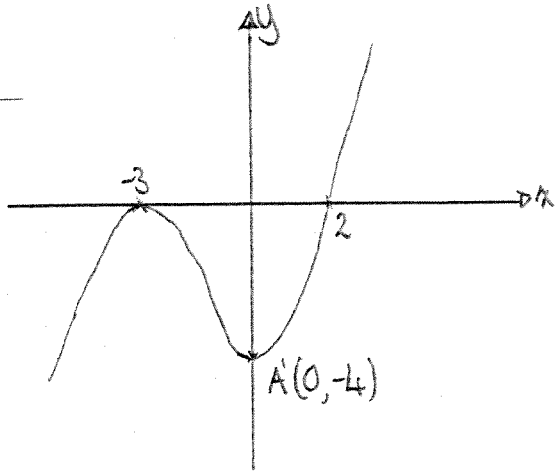
$$= \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \quad ✓$$

$$= \frac{4\sqrt{2}}{2}$$

$$= 2\sqrt{2} \quad ✓$$

(6)

(5)



✓ 3 units to left

✓ reflection in x-axis

✓ (-3,0) (2,0) (0,-4)

(3)

(6) (a) $f(x) = \frac{1}{x}(x^2 - \sqrt{x})$

$$= \frac{x^2}{x} - \frac{x^{\frac{1}{2}}}{x} \quad ✓$$

$$= x - x^{-\frac{1}{2}} \quad ✓$$

$$f'(x) = 1 + \frac{1}{2}x^{-\frac{3}{2}} \quad ✓$$

$$= 1 + \frac{1}{2x^{\frac{3}{2}}} \quad ✓$$

(4)

(b) $f'(x) = 5$

$$\Rightarrow \quad 1 + \frac{1}{2x^{\frac{3}{2}}} = 5 \quad ✓$$

$$\frac{1}{2x^{\frac{3}{2}}} = 4$$

$$\frac{1}{2} = 4x^{\frac{3}{2}}$$

$$x^{\frac{3}{2}} = \frac{1}{8} \quad ✓$$

$$x^3 = \frac{1}{64}$$

$$x = \frac{1}{4} \quad ✓$$

(3)

(7) (a) $Q(-2, 6)$ ✓ (1)

(b) $k = 6$ ✓ (1)

(c) $QC = 8 - (-2)$
$= 10$ ✓

or $CP^2 = (8-2)^2 + (6-2)^2$
$CP = 10$.

$$(x-a)^2 + (y-b)^2 = r^2 \quad ✓$$

$$(x-8)^2 + (y-6)^2 = 100. \quad ✓$$

(3)

(8) (a) $u_{n+1} = a u_n + 20$

$u_0 = 10$ and $u_1 = 26$

$$u_1 = a u_0 + 20$$

$$26 = 10a + 20 \quad ✓$$

$$10a = 6$$

$$a = \frac{3}{5} \quad ✓$$

(2)

(b) $u_2 = \frac{3}{5} u_1 + 20$

$= \frac{3}{5} \times 26 + 20$

$= \frac{78}{5} + \frac{100}{5}$

$= \frac{178}{5}$

$= 35\frac{3}{5}$ ✓

So $S_2 = u_1 + u_2$

$= 26 + 35\frac{3}{5}$

$= 61\frac{3}{5}$ ✓ (2)

(9) $\frac{dy}{dx} = 3x^2 - 4x$

Integrate

$y = \int (3x^2 - 4x)\,dx$ ✓

$y = \frac{3x^3}{3} - \frac{4x^2}{2} + c$

$y = x^3 - 2x^2 + c$ ✓

Substitute $(3, -7)$

$-7 = 27 - 2(9) + c$

$-7 = +9 + c$

$c = -16$ ✓

So $y = x^3 - 2x^2 - 16$ ✓ (4)

(10) (a) $f(\theta) = 4\cos^2 2\theta + 8\cos 2\theta + 6$

$f(\theta) = 4(\cos^2 2\theta + 2\cos 2\theta) + 6$ ✓

$= 4\left((\cos 2\theta + 1)^2 - 1^2\right) + 6$ ✓

$= 4(\cos 2\theta + 1)^2 + 2.$ ✓

$a = 4 \quad b = 1 \quad c = 2$ ✓ (4)

(b) minimum $0 + 2 = 2$. ✓

when $\cos 2\theta + 1 = 0$ ✓

$\cos 2\theta = -1$

$2\theta = \pi$

$\theta = \frac{\pi}{2}$. ✓

(3)