Fortrose Academy

Established 1791

Prelim Examination 2006 / 2007
(Assessing Units 1 & 2)

MATHEMATICS

Higher Grade - Paper I (Non~calculator)

Time allowed - 1 hour 10 minutes

Read Carefully

1. **Calculators may not be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

© Pegasys 2006

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

© Pegasys 2006

All questions should be attempted

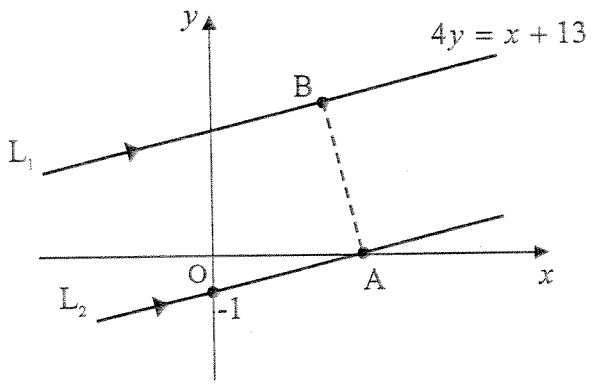
1. Two functions, defined on suitable domains, are given as

$$f(x) = x(x^2 - 1) \quad \text{and} \quad g(x) = x - 1.$$

(a) Show that the composite function, $h(x) = f(g(x))$, can be written in the form $h(x) = ax^3 + bx^2 + cx$, where a, b and c are constants, and state the value(s) of a, b and c. 4

(b) Hence solve the equation $h(x) = 6$, for x, showing clearly that there is only one solution. 4

2. Part of the line, L_1, with equation $4y = x + 13$, is shown in the diagram.
The line L_2 is parallel to L_1 and passes through the point (0,-1). Point A lies on the x-axis.



(a) Establish the equation of line L_2 and write down the coordinates of the point A. 3

(b) Given that the line AB is perpendicular to both lines, find, algebraically, the coordinates of point B. 5

(c) Hence calculate the **exact** shortest distance between the lines L_1 and L_2. 2

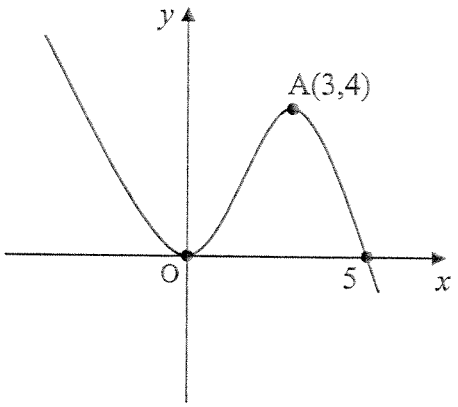
3. For what value of p, where $p > 0$, does the equation $(p^2 + 11)x^2 - 12px + p^2 = 0$ have equal roots? 6

© Pegasys 2006

4. Given that $\sin A = \frac{2}{\sqrt{6}}$ and $\cos B = \frac{\sqrt{2}}{\sqrt{3}}$, with angles A and B both being acute, show clearly that

$$3\cos(A - B) = 2\sqrt{2}.$$ 6

5. The diagram shows part of the graph of $y = f(x)$.



Sketch the graph of $y = -[f(x+3)]$ marking clearly the **new** positions of the highlighted points and stating their new coordinates. 3

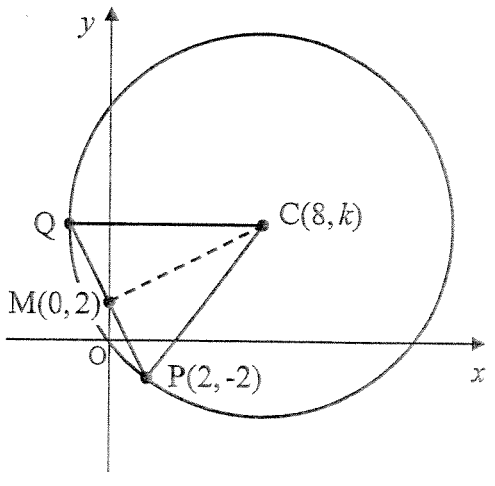
6. A function, f, is defined on a suitable domain as $f(x) = \frac{1}{x}\left(x^2 - \sqrt{x}\right)$.

(a) Differentiate f with respect to x, expressing your answer with positive indices. 4

(b) Hence find x when $f'(x) = 5$. 3

7. A circle, centre C$(8, k)$, has the points P(2,-2) and Q on its circumference as shown.

M(0,2) is the mid-point of the chord PQ.



(a) Find the coordinates of Q. 1

(b) Given that radius CQ is horizontal, write down the value of k, the y-coordinate of C. 1

(c) Hence establish the equation of the circle. 3

© Pegasys 2006

8. A sequence is defined by the recurrence relation $U_{n+1} = aU_n + 20$, where a is a constant.

(a) Given that $U_0 = 10$ and $U_1 = 26$, find a. 2

(b) Find the value of S_2, if $S_2 = U_1 + U_2$. 2

9. A curve has as its derivative $\frac{dy}{dx} = 3x^2 - 4x$.

Given that the point $(3, -7)$ lies on this curve, express y in terms of x. 4

10. A function is given as $f(\theta) = 4\cos^2 2\theta + 8\cos 2\theta + 6$ for $0 \leq \theta \leq \pi$.

(a) Express the function in the form $f(\theta) = a(\cos 2\theta + b)^2 + c$ and write down the values of a, b and c. 4

(b) Hence state the minimum value of this function and the corresponding replacement for θ. 3

[END OF QUESTION PAPER]

© Pegasys 2006