GCC Logs and Exponenetials

[SQA] 1. Evaluate $\log_5 2 + \log_5 50 - \log_5 4$. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	2	C	NC	A28	2	2000 P1 Q9
	1	A/B	NC	A28		

•[1] pd: use $\log_a x + \log_a y = \log_a xy$ •[2] pd: use $\log_a x - \log_a y = \log_a \frac{x}{y}$ •[3] pd: use $\log_a a = 1$	•[1] $\log_5 100 - \log_5 4$ •[2] $\log_5 25$ •[3] 2

2. (*a*) Given that $\log_4 x = P$, show that $\log_{16} x = \frac{1}{2}P$. **3**

(*b*) Solve $\log_3 x + \log_9 x = 12$. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	3	A	CN	A28	proof	2010 P2 Q7
(*b*)	3	A	CN	A32	$x = 3^8 (= 6561)$	

•[1] ss: convert from log to exponential form •[2] ss: know to and convert back to log form •[3] pd: process and complete •[4] ss: use appropriate strategy •[5] pd: start solving process •[6] pd: complete process via log to expo form	•[1] $x = 4^P$ •[2] $\log_{16} x = \log_{16} 4^P$ •[3] $\log_{16} x = P \times \log_{16} 4$ and complete •[4] $\log_3 x + \frac{1}{2}\log_3 x = 12$ •[5] $\log_3 x = 8$ •[6] $x = 3^8 (= 6561)$

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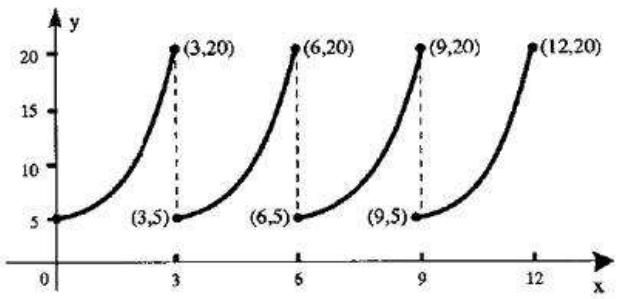
[SQA] 3. **Medical researchers studying the growth of a strain of bacteria observe that the number of bacteria, present after t hours, is given by the formula $N(t) = 40e^{1\cdot5t}$.**

(a) State the number of bacteria present at the start of the experiment. 1
(b) How many minutes will the bacteria take to double in number? 4

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	1	C	CR	A6		1989 P1 Q20
(*b*)	1	C	CR	A30		
(*b*)	3	A/B	CR	A30		

- •1 40
- •2 $40e^{1\cdot5t} = 80$
- •3 $1\cdot5t = \ln 2$
- •4 $t = 0.46$
- •5 28 minutes

[SQA] 4. **A medical technician obtains this print-out of a wave form generated by an oscilloscope. The technician knows that the equation of the first branch of the graph (for $0 \le x \le 3$) should be of the form $y = ae^{kx}$.**



(a) Find the values of a and k. 4

(b) Find the equation of the second branch of the curve (i.e. for $3 \le x \le 6$). 1

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	1	C	CR	A2		1993 P1 Q15
(*a*)	3	A/B	CR	A30		
(*b*)	1	A/B	CR	A7		

- •1 $(0,5) \Rightarrow a = 5$
- •2 $20 = 5e^{3k}$
- •3 *e.g.* $\ln 20 = \ln 5 + 3k \ln e$
- •4 $k = 0.462$ (Accept $\frac{1}{3}\ln 4$)
- •5 $y = 5e^{k(x-3)}$

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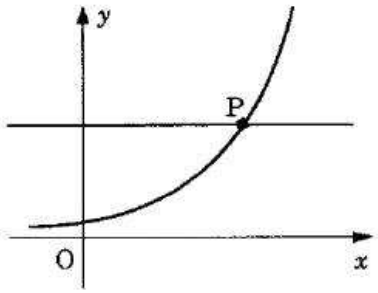
[SQA] 5. The diagram shows part of the graph with equation $y = 3^x$ and the straight line with equation $y = 42$. These graphs intersect at P.



Solve algebraically the equation $3^x = 42$, and hence write down, correct to 3 decimal places, the coordinates of P. 4

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	4	A/B	CR	A30		1994 P1 Q20

•1	use logs	*OR*	•1	use logs	*OR*	•1	use exponentials
•2	$\ln 3^x = \ln 42$		•2	$x = \log_3 42$		•2	$\left(e^{1\cdot0986}\right)^x = 42$
•3	$x \ln 3 = \ln 42$		•3	$x = \dfrac{\ln 42}{\ln 3}$		•3	$1\cdot0986x = \ln 42$
•4	$3\cdot402$		•4	$3\cdot402$		•4	$3\cdot402$

[SQA] 6. The amount A grams of a radioactive substance at time t minutes is given by $A = A_0e^{-kt}$ where A_0 is the initial amount of the substance and k is a constant.
In 3 minutes, 10 grams of the substance Bismuth are reduced to 9 grams through radioactive decay.

(*a*) Find the value of k. 3

The half-life of a substance is the length of time in which half the substance decays.
(*b*) Find the half-life of Bismuth. 2

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	1	C	CR	A30		1995 P1 Q18
(*a*)	2	A/B	CR	A30, A34		
(*b*)	2	A/B	CR	A30		

•1	$9 = 10e^{-3k}$
•2	$-3k = \log_e 0\cdot9$
•3	$0\cdot04$
•4	$e^{-kt} = 0\cdot5$
•5	a correct value for t

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[SQA] 7. A mug of tea cools according to the law $T_t = T_0 e^{-kt}$ where T_0 is the initial temperature and T_C is the temperature after t minutes. All temperatures are in °C.

(a) A particular mug of tea cooled from boiling point (100°C) to 75°C in a quarter of an hour. Calculate the value of k. 3

(b) By how many degrees will the temperature of this tea fall in the next quarter of an hour? 2

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	1	C	CR	A30		1996 P1 Q19
	4	A/B	CR	A30, A34		

- •1 $75 = 100e^{-k\times 15}$
- •2 $\ln 0{\cdot}75 = -15k$
- •3 $k = 0.0192$
- •4 $T_{15} = 75e^{-0.0192\times 15}$ *or* $T_{30} = 100e^{-0.0192\times 30}$
- •5 fall = 18.75

[SQA] 8. Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0 e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later.

If it takes one and a half hours for the area of the forest fire to double, find the value of the constant k. 3

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	3	A/B	CR	A30	$k = 0{\cdot}46$	2001 P2 Q9

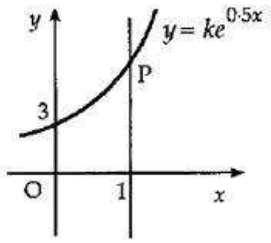
- •1 ic: form exponential equation
- •2 ss: express exp. equ. as log equation
- •3 pd: solve log equation

- •1 $2A_0 = A_0 e^{k\times 1{\cdot}5}$
- •2 e.g. $1{\cdot}5k = \ln 2$
- •3 $k = 0{\cdot}46$

[SQA] 9. The diagram shows part of the graph of $y = ke^{0.5x}$.

(a) Find the value of k. 1

(b) The line with equation $x = 1$ intersects the graph at P. Find the coordinates of the point P. 2



Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	1	C	CR	A30, A34		1991 P1 Q4
(b)	2	C	CR	A6		

- •1 $3 = ke^0 \Rightarrow k = 3$
- •2 $y = 3e^{0.5}$ or equivalent
- •3 $(1, 4{\cdot}9)$

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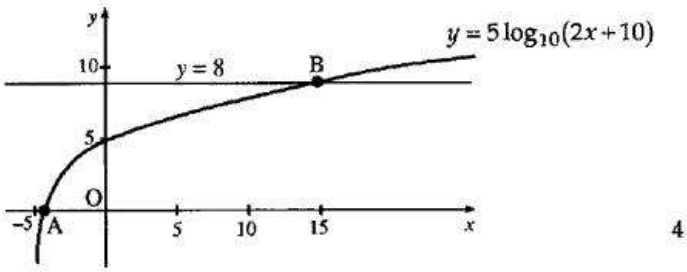
[SQA] 10. Part of the graph of $y = 5\log_{10}(2x+10)$ is shown in the diagram. This graph crosses the x-axis at the point A and the straight line $y = 8$ at the point B.

Find algebraically the x-coordinates of A and B. **4**



Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	4	A/B	CR	A31		1997 P1 Q17

- •1 $x_A = -4{\cdot}5$
- •2 $5\log_{10}(2x+10) = 8$
- •3 $2x+10 = 10^{\frac{8}{5}}$
- •4 $x = 14{\cdot}9$

[SQA] 11. Find the x-coordinate of the point where the graph of the curve with equation $y = \log_3(x-2)+1$ intersects the x-axis. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	2	C	CN	A31		2002 P2 Q7
	1	A/B	CN	A32	$x = 2\frac{1}{3}$	

- •1 ss: know to isolate log term
- •2 pd: express log equation as exp. equ.
- •3 pd: process

- •1 $\log_3(x-2) = -1$
- •2 $x - 2 = 3^{-1}$
- •3 $x = 2\frac{1}{3}$

[SQA] 12. Given $x = \log_5 3 + \log_5 4$, find algebraically the value of x. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	1	C	NC	A31		1998 P1 Q19
	3	A/B	NC	A28		

- •1 $x = \log_5 12$
- •2 $5^x = 12$
- •3 $\log 5^x = \log 12$
- •4 $\frac{\log_{10} 12}{\log_{10} 5}$ *or* $\frac{\log_e 12}{\log_e 5}$ *or* $\frac{\log 12}{\log 5} = 1.54$

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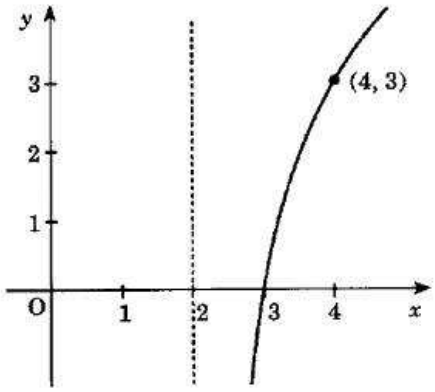
[SQA] 13. The diagram shows a sketch of the graph of $y = f(x)$ where $f(x) = a\log_2(x-b)$.
Find the values of a and b.



3

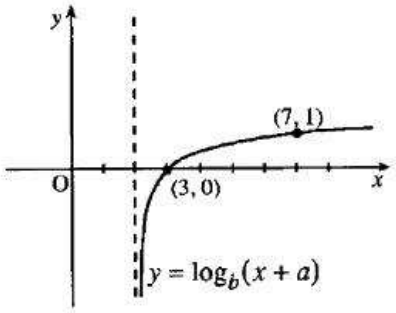
Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	3	A/B	CN	A31, A2		1995 P1 Q19

$\bullet^1$ $b = 2$

$\bullet^2$ $3 = a\log_2 2$ stated or implied
or $(4-b)^a = 8$

$\bullet^3$ $a = 3$

[SQA] 14. The diagram shows part of the graph of $y = \log_b(x+a)$.
Determine the values of a and b.



3

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	3	A/B	CN	A31, A2		1999 P1 Q15

$\bullet^1$ $a = -2$
$\bullet^2$ $1 = \log_b(7-2)$
$\bullet^3$ $b = 5$

OR

$\bullet^1$ $1 = \log_b(7+a)$ **and** $0 = \log_b(a+3)$
$\bullet^2$ $7 + a = b$ **and** $a + 3 = b^0$
$\bullet^3$ $a = -2,\ b = 5$

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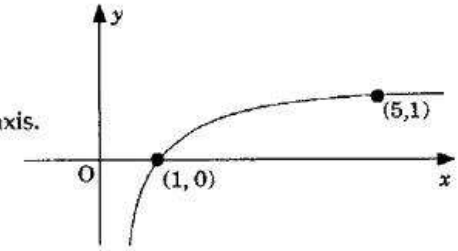
[SQA] 15. The diagram shows a sketch of part of the graph of $y = \log_5 x$.

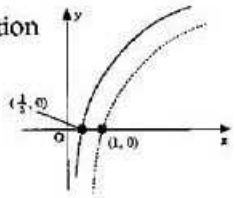
(*a*) Make a copy of the graph of $y = \log_5 x$.
On your copy, sketch the graph of $y = \log_5 x + 1$.
Find the coordinates of the point where it crosses the x-axis. 3



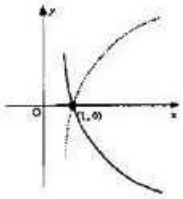
(*b*) Make a second copy of the graph of $y = \log_5 x$.
On your copy, sketch the graph of $y = \log_5 \frac{1}{x}$. 2

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	2	C	NC	A32		1994 P1 Q16
(*a*)	1	A/B	NC	A3		
(*b*)	2	A/B	NC	A28, A3		

•1 sketch of new function
•2 $\log_5 x + 1 = 0$
•3 $\left(\frac{1}{5}, 0\right)$



•4 $\log_5 \frac{1}{x} = -\log_5 x$
•5 reflect in x-axis



[SQA] 16.

(*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.
(ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4**

(*b*) Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$. **5**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	4	C	CN	A21	$(x-1)(x+4)(x+5)$	2009 P2 Q3
(*b*)	5	B	CN	A32	$x = 1$	

•1 ss: know and use $f(a) = 0 \Leftrightarrow a$ is a root
•2 ic: start to find quadratic factor
•3 ic: complete quadratic factor
•4 pd: factorise fully

•5 ss: use log laws
•6 ss: know to & convert to exponential form
•7 ic: write cubic in standard form
•8 pd: solve cubic
•9 ic: interpret valid solution

•1 $f(1) = 1 + 8 + 11 - 20 = 0$ so $x = 1$ is a root
•2 $(x-1)(x^2 \cdots$
•3 $(x-1)(x^2 + 9x + 20)$
•4 $(x-1)(x+4)(x+5)$

•5 $\log_2\big((x+3)(x^2+5x-4)\big)$
•6 $(x+3)(x^2+5x-4) = 2^3$
•7 $x^3 + 8x^2 + 11x - 20 = 0$
•8 $x = 1$ or $x = -4$ or $x = -5$
•9 $x = 1$ only

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[SQA] 17. Find x if $4\log_x 6 - 2\log_x 4 = 1$. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	3	C	NC	A32, A28, A31	$x = 81$	2001 P1 Q8

•[1] pd: use log-to-index rule •[2] pd: use log-to-division rule •[3] ic: interpret base for $\log_x a = 1$ and simplify	•[1] $\log_x 6^4 - \log_x 4^2$ •[2] $\log_x \frac{6^4}{4^2}$ •[3] all processing leading to $x = 81$

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[SQA] 18. When the switch in this circuit was closed, the computer printed out a graph of the current flowing (I microamps) against the time (t seconds).This graph is shown in fig. 1.

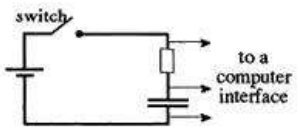
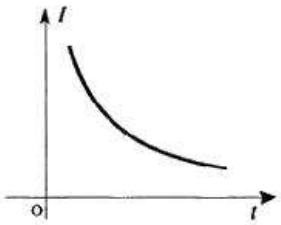


figure 1



In order to determine the equation of the graph shown in figure 1, values of $\log_e I$ were plotted against $\log_e t$ and the best fitting straight line was drawn as shown in figure 2.

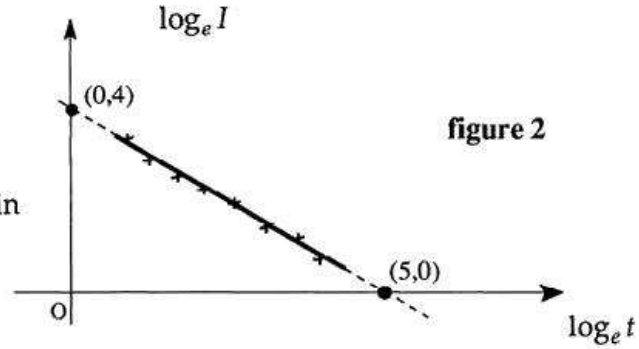


figure 2

(*a*) Find the equation of the line shown in figure 2 in terms of $\log_e I$ and $\log_e t$. **(3)**

(*b*) Hence or otherwise show that I and t satisfy a relationship of the form $I = kt^r$ stating the values of k and r. **(4)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	3	C	CR	G2, G3		1993 P2 Q10
(*b*)	4	A/B	CR	A33		

(*a*) •1 $m = -\frac{4}{5}$ stated or implied

•2 $y = mx + 4$ stated or implied

•3 $\log_e I = -\frac{4}{5}\log_e t + 4$

(*b*) •4 $\log_e t^{-\frac{4}{5}}$

•5 $\log_e 54.6$

•6 $\log_e 54.6t^{-\frac{4}{5}}$

•7 $I = 54.6t^{-0.8}$

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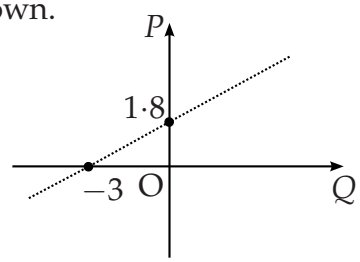
[SQA] 19. The results of an experiment give rise to the graph shown.



(*a*) Write down the equation of the line in terms of P and Q. **2**

It is given that $P = \log_e p$ and $Q = \log_e q$.

(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	2	A/B	CR	G3	$P = 0{\cdot}6Q + 1{\cdot}8$	2000 P2 Q11
(*b*)	4	A/B	CR	A33	$a = 6{\cdot}05$, $b = 0{\cdot}6$	

$\bullet^1$ ic: interpret gradient
$\bullet^2$ ic: state equ. of line

$\bullet^3$ ic: interpret straight line
$\bullet^4$ ss: know how to deal with x of $x \log y$
$\bullet^5$ ss: know how to express number as log
$\bullet^6$ ic: interpret sum of two logs

$\bullet^1$ $m = \frac{1{\cdot}8}{3} = 0{\cdot}6$
$\bullet^2$ $P = 0{\cdot}6Q + 1{\cdot}8$

Method 1

$\bullet^3$ $\log_e p = 0{\cdot}6 \log_e q + 1{\cdot}8$
$\bullet^4$ $\log_e q^{0{\cdot}6}$
$\bullet^5$ $\log_e 6{\cdot}05$
$\bullet^6$ $p = 6{\cdot}05q^{0{\cdot}6}$

Method 2
$\ln p = \ln aq^b$

$\bullet^3$ $\ln p = \ln a + b \ln q$
$\bullet^4$ $\ln p = 0{\cdot}6 \ln q + 1{\cdot}8$ *stated or implied by* $\bullet^5$ *or* $\bullet^6$
$\bullet^5$ $\ln a = 1{\cdot}8$
$\bullet^6$ $a = 6{\cdot}05$, $b = 0{\cdot}6$

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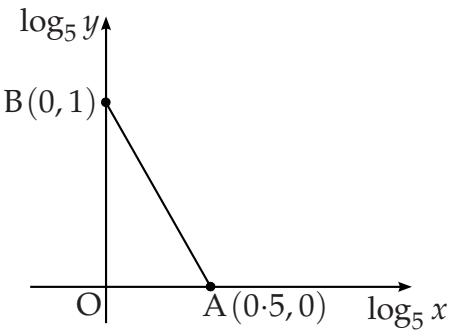
[SQA] 20. The graph illustrates the law $y = kx^n$.

If the straight line passes through $A(0{\cdot}5, 0)$ and $B(0, 1)$, find the values of k and n. **4**



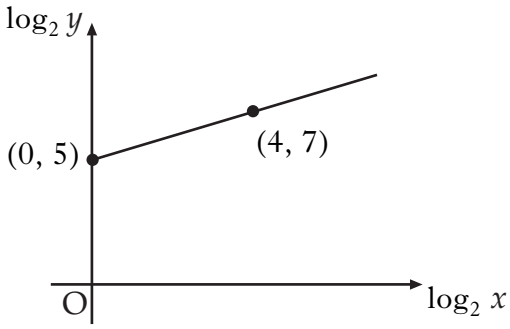
Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	4	A/B	NC	A33	$y = 5x^{-2}$	2002 P1 Q11

•1 ic: interpret graph •2 ss: use log laws •3 ss: use log laws •4 pd: solve log equation	•1 $\log_5 y = -2(\log_5 x) + 1$ •2 $\log_5 y = \log_5 x^{-2} + \ldots$ •3 $\ldots + \log_5 5$ •4 $y = 5x^{-2}$

21. Variables x and y are related by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points $(0, 5)$ and $(4, 7)$, as shown in the diagram.

Find the values of k and n. **5**



Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	5	A	CN	A33	$k = 32, n = \frac{1}{2}$	2011 P2 Q5

•1 ss: introduce logarithms to $y = kx^n$ •2 ic: use laws of logarithms •3 ic: interpret intercept •4 ic: solve for k •5 ic: interpret gradient	•1 $\log_2 y = \log_2 kx^n$ •2 $\log_2 y = n \log_2 x + \log_2 k$ •3 $\log_2 k = 5$ or $\log_2 y = 5$ •4 $k = 32$ or 2^5 •5 $n = \frac{1}{2}$

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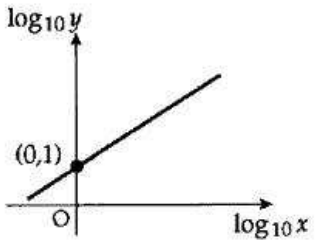
[SQA] 22. As shown in the diagram, a set of experimental results gives a straight line graph when $\log_{10} y$ is plotted against $\log_{10} y$. The straight line passes through (0, 1) and has a gradient of 2.

Express y in terms of x. 6



Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	2	C	CN	G3		1990 P1 Q14
	4	A/B	CN	A33, A34		

- •1 *use* $y = mx + c$
- •2 $\log_{10} y = 2\log_{10} x + 1$
- •3 $\log_{10} y = 2\log_{10} x + \log_{10} 10$
- •4 $\log_{10} y = \log_{10} x^2 + 1$
- •5 $\log_{10} y = \log_{10} 10x^2$
- •6 $y = 10x^2$

[END OF QUESTIONS]

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