GCC Logs and Exponenetials

[SQA] 1. Evaluate $\log_5 2 + \log_5 50 - \log_5 4$. **3**

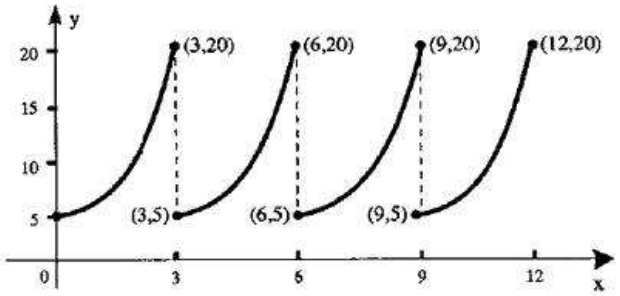
2. (*a*) Given that $\log_4 x = P$, show that $\log_{16} x = \frac{1}{2}P$. **3**

(*b*) Solve $\log_3 x + \log_9 x = 12$. **3**

[SQA] 3. Medical researchers studying the growth of a strain of bacteria observe that the number of bacteria, present after t hours, is given by the formula $N(t) = 40e^{1.5t}$.

(*a*) State the number of bacteria present at the start of the experiment. **1**
(*b*) How many minutes will the bacteria take to double in number? **4**

[SQA] 4. A medical technician obtains this print-out of a wave form generated by an oscilloscope. The technician knows that the equation of the first branch of the graph (for $0 \le x \le 3$) should be of the form $y = ae^{kx}$.

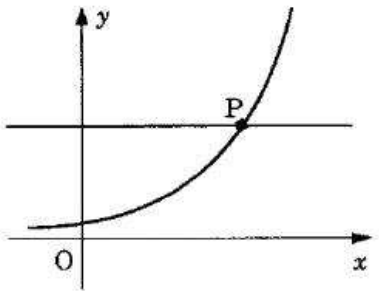


(*a*) Find the values of a and k. **4**

(*b*) Find the equation of the second branch of the curve (i.e. for $3 \le x \le 6$). **1**

[SQA] 5. The diagram shows part of the graph with equation $y = 3^x$ and the straight line with equation $y = 42$. These graphs intersect at P.

Solve algebraically the equation $3^x = 42$, and hence write down, correct to 3 decimal places, the coordinates of P. **4**



[SQA] 6. The amount A grams of a radioactive substance at time t minutes is given by $A = A_0e^{-kt}$ where A_0 is the initial amount of the substance and k is a constant.
In 3 minutes, 10 grams of the substance Bismuth are reduced to 9 grams through radioactive decay.

(*a*) Find the value of k. **3**

The half-life of a substance is the length of time in which half the substance decays.
(*b*) Find the half-life of Bismuth. **2**

[SQA] 7. A mug of tea cools according to the law $T_t = T_0 e^{-kt}$ where T_0 is the initial temperature and T_C is the temperature after t minutes. All temperatures are in °C.

(a) A particular mug of tea cooled from boiling point (100°C) to 75°C in a quarter of an hour. Calculate the value of k. 3

(b) By how many degrees will the temperature of this tea fall in the next quarter of an hour? 2

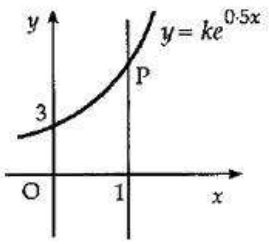
[SQA] 8. Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0 e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later.

If it takes one and a half hours for the area of the forest fire to double, find the value of the constant k. 3

[SQA] 9. The diagram shows part of the graph of $y = ke^{0.5x}$.

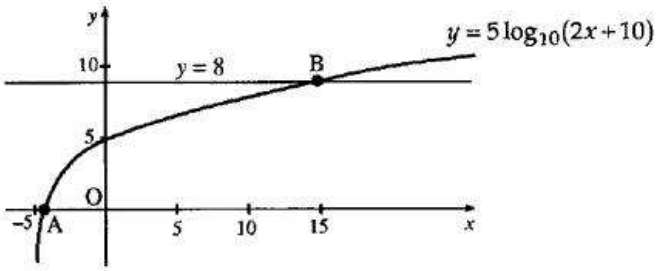
(a) Find the value of k. 1

(b) The line with equation $x = 1$ intersects the graph at P. Find the coordinates of the point P. 2



[SQA] 10. Part of the graph of $y = 5\log_{10}(2x+10)$ is shown in the diagram. This graph crosses the x-axis at the point A and the straight line $y = 8$ at the point B.

Find algebraically the x-coordinates of A and B. 4

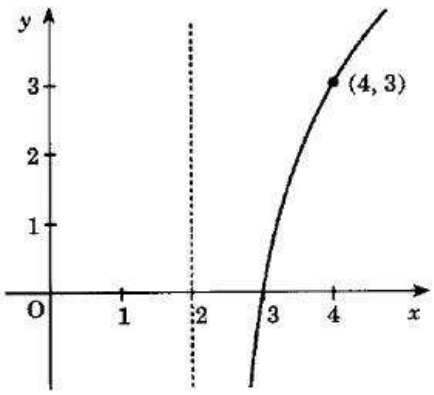


[SQA] 11. Find the x-coordinate of the point where the graph of the curve with equation $y = \log_3(x-2)+1$ intersects the x-axis. 3

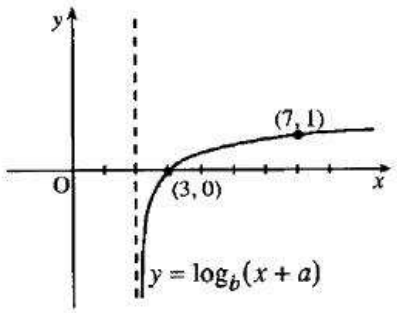
[SQA] 12. Given $x = \log_5 3 + \log_5 4$, find algebraically the value of x. 4

[SQA] 13. The diagram shows a sketch of the graph of $y = f(x)$ where $f(x) = a\log_2(x-b)$.

Find the values of a and b. 3



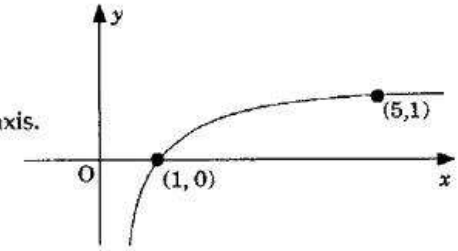
[SQA] 14. The diagram shows part of the graph of $y = \log_b(x+a)$.
Determine the values of a and b. 3



[SQA] 15. The diagram shows a sketch of part of the graph of $y = \log_5 x$.

(*a*) Make a copy of the graph of $y = \log_5 x$.
On your copy, sketch the graph of $y = \log_5 x + 1$.
Find the coordinates of the point where it crosses the x-axis. 3

(*b*) Make a second copy of the graph of $y = \log_5 x$.
On your copy, sketch the graph of $y = \log_5 \frac{1}{x}$. 2



[SQA] 16.

(*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.
(ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4**

(*b*) Solve $\log_2(x+3) + \log_2(x^2 + 5x - 4) = 3$. **5**

[SQA] 17. Find x if $4\log_x 6 - 2\log_x 4 = 1$. **3**

[SQA] 18. When the switch in this circuit was closed, the computer printed out a graph of the current flowing (I microamps) against the time (t seconds).This graph is shown in fig. 1.

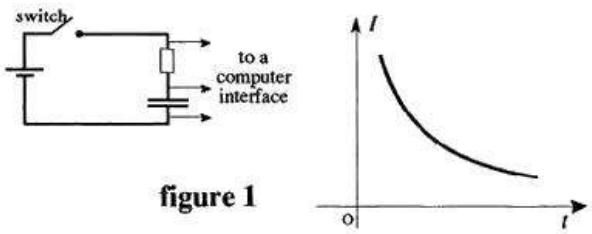


figure 1

In order to determine the equation of the graph shown in figure 1, values of $\log_e I$ were plotted against $\log_e t$ and the best fitting straight line was drawn as shown in figure 2.

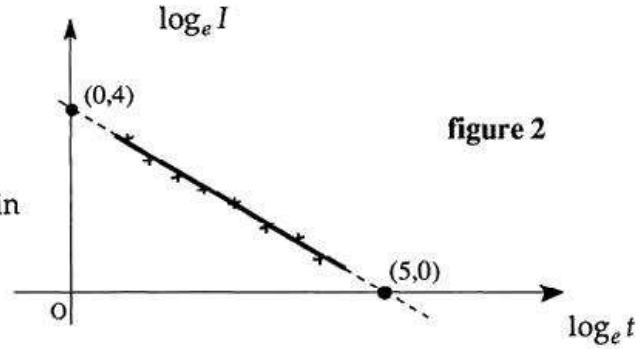
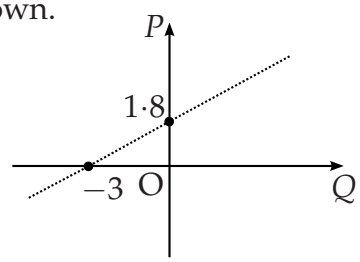


figure 2

(*a*) Find the equation of the line shown in figure 2 in terms of $\log_e I$ and $\log_e t$. **(3)**

(*b*) Hence or otherwise show that I and t satisfy a relationship of the form $I = kt^r$ stating the values of k and r. **(4)**

[SQA] 19. The results of an experiment give rise to the graph shown.



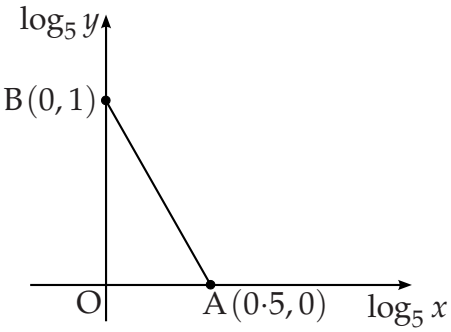
(*a*) Write down the equation of the line in terms of P and Q. **2**

It is given that $P = \log_e p$ and $Q = \log_e q$.

(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. **4**

[SQA] 20. The graph illustrates the law $y = kx^n$.

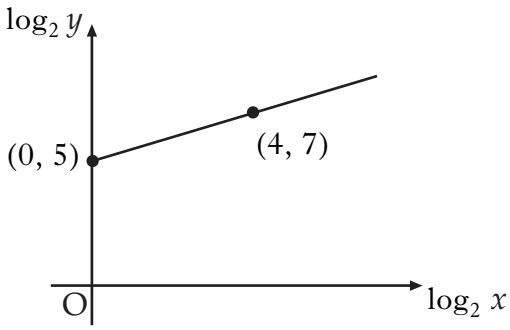
If the straight line passes through $A(0{\cdot}5, 0)$ and $B(0, 1)$, find the values of k and n. **4**



21. Variables x and y are related by the equation $y = kx^n$.

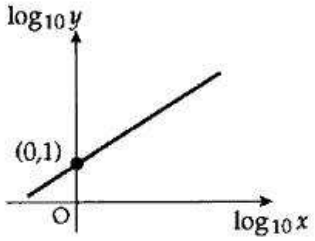
The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points $(0, 5)$ and $(4, 7)$, as shown in the diagram.

Find the values of k and n. **5**



[SQA] 22. As shown in the diagram, a set of experimental results gives a straight line graph when $\log_{10} y$ is plotted against $\log_{10} y$. The straight line passes through (0, 1) and has a gradient of 2.

Express y in terms of x. **6**



[END OF QUESTIONS]