Advanced Higher Functions.

NAB Practice Questions.

① $f(x)=\dfrac{x^2+6x+12}{x+2}$, $x \neq -2$, $x \in \Re$.

(a) Write down the equation of the vertical asymptote of the graph of $y=f(x)$. (1)

(b) Show that the graph has a non-vertical asymptote and find its equation. (2)

(c) Sketch the graph of $y=f(x)$ showing clearly its intersections with the axes, and its turning points with appropriate justification. (6)

② $f(x)=\dfrac{2x^2+7x+3}{(x-3)(x+1)}$, $x \neq -1, 3$, $x \in \Re$.

(a) Write down the equations of the vertical asymptotes of the graph of $y=f(x)$. (2)

(b) Show that the graph has a horizontal asymptote and find its equation. (2)

(c) Sketch the graph of $y=f(x)$ showing clearly its intersections with the axes, and its turning points with appropriate justification. (6)

Functions

(1) $y = \dfrac{x^2+6x+12}{x+2}$

Vertical asymptote $x = -2$.

approach

x	$\to$	-2	$\to$
$y = \dfrac{x^2+6x+12}{x+2}$	$\dfrac{+}{-}$	∞	$\dfrac{+}{+}$
	$-$	∞	$+$

2

Horizontal Asymptotes / Oblique

$$y = \frac{x^2+6x+12}{x+2}$$

$$= x+4+\frac{4}{x+2}$$

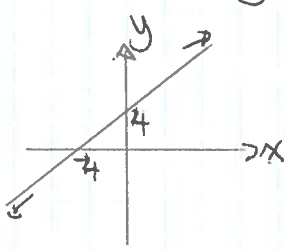
$$\begin{array}{r|l} & x+4 \\ \hline x+2 & x^2+6x+12 \\ & x^2+2x \\ \hline & 4x+12 \\ & 4x+8 \\ \hline & 4 \end{array}$$

As $x \to \pm\infty$ $y \to x+4$ so $y = x+4$ is an oblique asymptote.

approach as $x \to +\infty$ $y \to x+4$ + a bit above.

as $x \to -\infty$ $y \to x+4$ − a bit below

3



cuts x-axis $y=0$

$\Rightarrow x^2+6x+12=0$

$b^2-4ac = 36-4\times 12$
$= -ve.$

doesn't cut.

cuts y-axis $x=0$ $\Rightarrow y=6$ $(0,6)$

Stationary Points

$$y = x + 4 + \frac{4}{x+2}$$

$$= x + 4 + 4(x+2)^{-1}$$

$$\frac{dy}{dx} = 1 - 4(x+2)^{-2}$$

$$= 1 - \frac{4}{(x+2)^2}.$$

For stationary points $\frac{dy}{dx} = 0$

$$\Rightarrow 1 - \frac{4}{(x+2)^2} = 0$$

$$\frac{4}{(x+2)^2} = 1$$

$$(x+2)^2 = 4$$

$$x + 2 = \pm 2.$$

$$x = 0 \text{ or } -4.$$

$\Rightarrow y = 6$ or $y = -2.$ $(0,6)$ $(-4,-2)$.

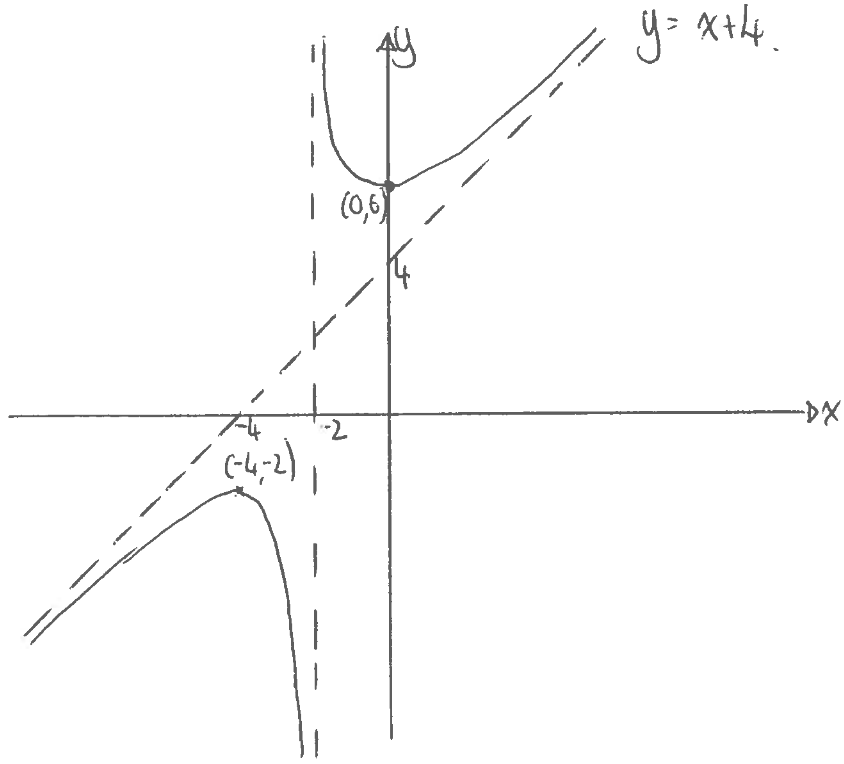
Nature

$$\frac{dy}{dx} = 1 - 4(x+2)^{-2}$$

$$\frac{d^2y}{dx^2} = 8(x+2)^{-3} \quad \frac{8}{(x+2)^3}$$

When $x = 0$ $\frac{d^2y}{dx^2} = 1$ > 0 so $(0,6)$ is a minimum TP.

When $x = -4$ $\frac{d^2y}{dx^2} = -1$ < 0 so $(-4,-2)$ is a maximum TP.



ii

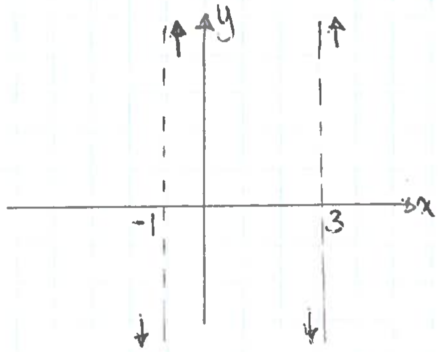
(2) $y = \dfrac{2x^2+7x+3}{(x-3)(x+1)}$

<u>vertical asymptotes</u> $x=3$ and $x=-1$

approach.

x	$\to$	3	$\to$
$y = \dfrac{2x^2+7x+3}{(x-3)(x+1)} = \dfrac{(2x+1)(x+3)}{(x-3)(x+1)}$	$\dfrac{(+)(+)}{(-)(+)}$	∞	$\dfrac{(+)(+)}{(+)(+)}$
	$-$	∞	$+$

x	$\to$	-1	$\to$
$y = \dfrac{(2x+1)(x+3)}{(x-3)(x+1)}$	$\dfrac{(-)(+)}{(-)(-)}$	∞	$\dfrac{(-)(+)}{(-)(+)}$
	$-$	∞	$+$.



(4)

<u>horizontal asymptotes.</u>

$$y = \frac{2x^2+7x+3}{x^2-2x-3}$$

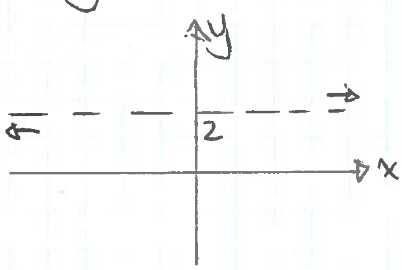
$$\begin{array}{r} 2 \\ x^2-2x-3 \;\big)\; 2x^2+7x+3 \\ \underline{2x^2-4x-6} \\ 11x+9 \end{array}$$

$$y = 2 + \frac{11x+9}{x^2-2x-3}$$

As $x \to \pm\infty$ $y \to 2$ so $y=2$ is a horizontal asymptote

<u>Approach</u> as $x \to \infty$ $y \to 2$ + a little bit above

$x \to -\infty$ $y \to 2$ − a little bit below



(4)

Cuts x-axis $\Rightarrow$ $y=0$

$$(2x+1)(x+3)=0$$

$$x=-\tfrac{1}{2} \text{ or } x=-3.$$

$(-\tfrac{1}{2},0)$ $(-3,0)$

Cuts y-axis $\Rightarrow$ $x=0$ $y=\frac{3}{-3}=-1$ $(0,-1)$.

Stationary Points.

$$y = 2+\frac{11x+9}{x^2-2x-3}$$

$u=11x+9$ $\frac{du}{dx}=11$

$v=x^2-2x-3$ $\frac{dv}{dx}=2x-2$.

$$\frac{dy}{dx} = \frac{(x^2-2x-3).11-(11x+9)(2x-2)}{(x^2-2x-3)^2}$$

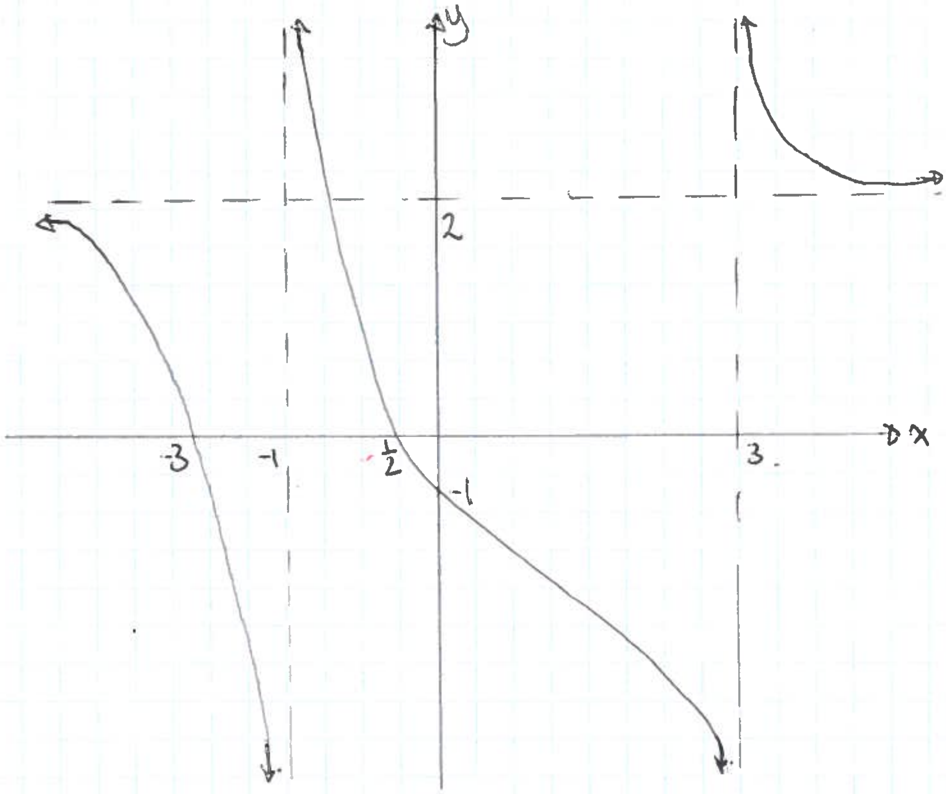
$$= \frac{11x^2-22x-33-22x^2+4x+18}{(x^2-2x-3)^2}.$$

For stationary points $\frac{dy}{dx}=0$

$$\Rightarrow -11x^2-18x-15=0.$$

$$b^2-4ac = (-18)^2-4(-11)(-15)$$

$$= -ve. \quad \therefore \text{ no stationary points.}$$



8

Advanced Higher 1 **Assignment**

Functions

1. Prove whether the following functions are odd , even or neither.

(a) $f(x) = x^6 - 6x^2 - 1$ (b) $f(x) = 3\sin x$ **2, 2**

2. Find the domain and range of the following functions

(a) $f(x) = \sqrt{x-3}$ (b) $f(x) = \dfrac{2}{\cos x}$ **2,2**

3. $f(x) = 3x - 1$

Sketch the graph of $f(x) = |3x - 1|$ **3**

4. $y = x + 2 + \dfrac{1}{x-1}$

(a) Write down the equation of the vertical asymptote and the coordinates of the y-intercept.

(b) Find the equation of the non-vertical asymptote.

(c) Find the coordinatates and justify the nature of any stationary points.

(d) Sketch the curve showing all the main features. **2,2,4,2**

5. $f(x) = \dfrac{x^2 - x - 5}{x+2}, \quad x \neq -2$

(a) Write down the equation of the vertical asymptote and the coordinates of the y- intercept.

(b) Find the equation of the non-vertical asymptote.

(c) Find the coordinates and justify the nature of any stationary points.

(d) Sketch the curve showing all the main features.

2,3,4,2

Total = 32 marks

Functions AH1 Assignment

① (a) $f(x) = x^6 - 6x^2 - 1$

$f(-x) = x^6 - 6x^2 - 1$

$f(x) = f(-x)$ so even

(b) $f(x) = 3\sin x$

$f(-x) = -3\sin x$

$= -f(x)$

so odd.

② (a) $f(x) = \sqrt{x-3}$

Domain

$x \geq 3$

Range

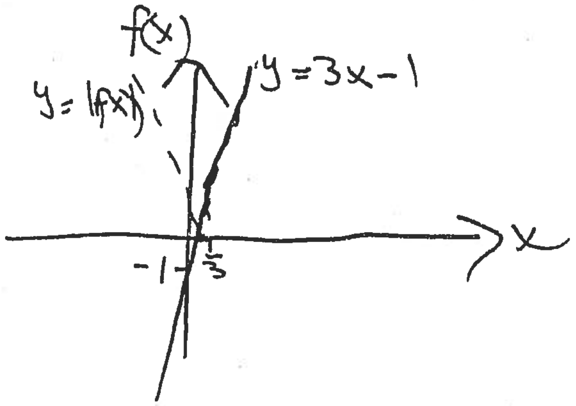
$f(x) \geq 0$

(b) $f(x) = \frac{2}{\cos x}$

Domain $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Range $-2 \leq f(x) \leq 2$

③ $f(x) = 3x - 1$



④ $y = x + 2 + \frac{1}{(x-1)}$

$\Rightarrow y = x + 2 + (x-1)^{-1}$

(a) V.A. $x = 1$

y-intercept $x = 0$

$y = 1$

(b) Non-vertical asymptote $y = x + 2$

($y \to x+2$ as $x \to 1$)

(c) $\frac{dy}{dx} = 1 - (x-1)^{-2}$

$= 0$ when $1 = \frac{1}{(x-1)^2}$

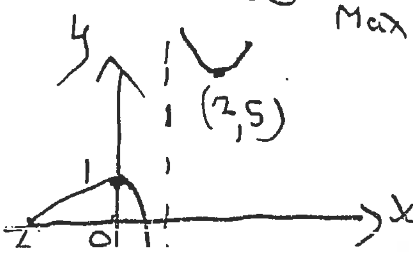
So $(x-1)^2 = 1$

$(x-1) = \pm 1$

$\frac{d^2y}{dx^2} = \frac{2}{(x-1)^3}$

$x = 2$ > 0 So min

$x = 0$ < 0 Max



(5) $f(x) = \left(\dfrac{x^2 - x - 5}{(x+2)}\right)$, $x \neq -2$

(a) V.A $x = -2$

y intecept $x = 0$, $y = -\frac{5}{2}$

(b)

$$\begin{array}{r|l} & x - 3 \\ \hline x+2 & x^2 - x - 5 \\ & -(x^2 + 2x) \\ \hline & -3x - 5 \\ & -(-3x - 6) \\ \hline & 1 \end{array}$$

, so $f(x) = x - 3 + \dfrac{1}{(x+2)}$,

Non-vertical asymptote $\underline{y = x - 3}$

(c) $y = (x-3) + (x+2)^{-1}$

$\dfrac{dy}{dx} = 1 - (x+2)^{-2}$

$\dfrac{dy}{dx} = 0$ when $1 = \dfrac{1}{(x+2)^2}$

$(x+2)^2 = 1$

$\Rightarrow x + 2 = \pm 1$

Stat pts $x = -1$ $(-1, -3)$ Min

$x = -3$ $(-3, -7)$ Max

$\dfrac{d^2y}{dx^2} = \dfrac{2}{(x+2)^3}$ $x = -1$, > 0 Min

$x = -3$, < 0 Max

(d)