AH Differentiation Homework (3)

1. A curve has equation $xy + y^2 = 2$.
   (a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y. (3)
   (b) Hence find an equation of the tangent to the curve at the point (1,1). (2)

2. Given the equation $2y^2 - 2xy - 4y + x^2 = 0$ of a curve, obtain the x-coordinate of each point at which the curve has a horizontal tangent. (4)

3. The equation $y^3 + 3xy = 3x^2 - 5$ defines a curve passing through the point A(2,1). Obtain an equation for the tangent to the curve at A. (4)

4. (a) Given $f(x) = x(1+x)^{10}$, obtain $f'(x)$ and simplify your answer. (3)

   (b) Given $\text{y} = 3^x$, use logarithmic differentiation to obtain $\frac{dy}{dx}$ in terms of x. (3)

5. (a) Given that $f(x) = \sqrt{x}\, e^{-x}$, $x \geq 0$, obtain and simplify $f\ '(x)$. (4)

   (b) Given $y = (x+1)^2(x+2)^{-4}$ and $x > 0$, use logarithmic differentiation to show that $\frac{dy}{dx}$ can be expressed in the form $\left(\frac{a}{x+1} + \frac{b}{x+2}\right)y$, stating the values of the constants a and b. (3)

6. A curve is defined by the equations

$$x = 5\cos\theta\ , \qquad y = 5\sin\theta, \qquad (0 \leq \theta \leq 2\pi).$$

   Use parametric differentiation to find $\frac{dy}{dx}$ in terms of θ. (2)

   Find the equation of the tangent to the curve at the point where $\theta = \frac{\pi}{4}$. (3)

7. A curve is defined by the parametric equations

$$x = t^2 + t - 1\ , \quad y = 2t^2 - t + 2$$

   for all t. Show that the point A(-1,5) lies on the curve and obtain an equation of the tangent to the curve at the point A. (6)

8. Part of a journey an object made was observed.

The displacement, s metres, of the object travelling in a straight line at time t seconds is given by :-

$$s = \frac{t^3}{3} + t^2 - 8t + 10$$

(a) How far from the origin was the object when the observation was started ?
(b) At what time was the object stationary ?
(c) Comment on the motion of the object when $t = 5\ secs$.
(d) Does the object ever reach a constant velocity or decelerate during its journey ? Justify your answer.

9. A bird drops a berry that falls along the path $y = -x^2 + 10 \quad (metres\ above\ the\ ground)$.

If $x = \frac{1}{2}t$ metres, with t in seconds, what is the velocity of the berry when it hits the ground?

(Remember to give magnitude and direction).

10. If $= \frac{\sin x}{x^2}$, prove that $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2 + 2)y = 0.$

11. Determine the stationary points of $y = x^3e^{-x}$. Use the second derivative to help determine the nature of the stationary points.