AH Differentiation 2

1) $\sec x = \dfrac{1}{\cos x}$ $\operatorname{cosec} x = \dfrac{1}{\sin x}$ $\cot x = \dfrac{1}{\tan x}$ ✓ 1

(b) $\dfrac{d}{dx}(\sec x)$

$= \dfrac{d}{dx}\left((\cos x)^{-1}\right)$

$= -(\cos x)^{-2} \cdot -\sin x$

$= \dfrac{\sin x}{\cos^2 x}$

$= \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x}$

$= \underline{\underline{\sec x \tan x}}$ ✓

$\dfrac{d}{dx}(\operatorname{cosec} x)$

$= \dfrac{d}{dx}\left(\dfrac{1}{\sin x}\right)$

$= \dfrac{0 \cdot \sin x - 1 \cdot \cos x}{\sin^2 x}$

$= -\dfrac{\cos x}{\sin^2 x}$

$= -\dfrac{\cos x}{\sin x} \cdot \dfrac{1}{\sin x}$ ✓

$= \underline{\underline{-\cot x \operatorname{cosec} x}}$

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$\dfrac{d}{dx}(\cot x)$

$= \dfrac{d}{dx}\left(\dfrac{\cos x}{\sin x}\right)$

$= \dfrac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$

$= \dfrac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$= -\dfrac{1}{\sin^2 x}$

$= \underline{\underline{-\operatorname{cosec}^2 x}}$ ✓

(c)(i) $\dfrac{d}{dx}(\cot 3x)$

$= \underline{\underline{-3\operatorname{cosec}^2 3x}}$ ✓

(ii) $\dfrac{d}{dx}(\operatorname{cosec} x)^2$

$= 2\operatorname{cosec} x \cdot (-\cot x \operatorname{cosec} x)$

$= \underline{\underline{-2\cot x \operatorname{cosec}^2 x}}$ ✓

(iii) $\dfrac{d}{dx}(2x^2 \sec x)$

$= 4x \sec x + 2x^2 \sec x \tan x$

$= \underline{\underline{2x \sec x (2 + x \tan x)}}$ ✓

/3

2) (a) $h(x) = -\ln(\cos 5x)$

$$h'(x) = -\frac{1}{\cos 5x} \cdot \frac{d}{dx}(\cos 5x)$$

$$= -\frac{1}{\cos 5x} \cdot (-5\sin 5x)$$

$$= \frac{5\sin 5x}{\cos 5x}$$

$$= \underline{\underline{5\tan 5x}}$$

/3

(b) $y = \frac{\ln(x+3)}{x+3}$

$$\frac{dy}{dx} = \frac{\frac{1}{x+3}(x+3) - \ln(x+3) \cdot 1}{(x+3)^2}$$

Quotient rule

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$$= \frac{1 - \ln(x+3)}{(x+3)^2}$$

3) (a) $y = x^2 \tan^{-1}(\sqrt{x-1})$

$U = x^2 \quad V = \tan^{-1}(x-1)^{\frac{1}{2}}$

$U' = 2x \quad V' = \frac{1}{1+(x-1)} \cdot \frac{1}{2}(x-1)^{-\frac{1}{2}}$

$= \frac{1}{x} \cdot \frac{1}{2\sqrt{x-1}}$

$= \frac{1}{2x\sqrt{x-1}}$

$$\frac{dy}{dx} = 2x \cdot \tan^{-1}(\sqrt{x-1}) + x^2 \cdot \frac{1}{2x\sqrt{x-1}}$$

product rule

$$= 2x\tan^{-1}(\sqrt{x-1}) + \frac{x}{2\sqrt{x-1}}$$

$$= \underline{\underline{x\left(2\tan^{-1}(\sqrt{x-1}) + \frac{1}{2}\sqrt{x-1}\right)}}$$

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3(b) $y = \tan^{-1} e^{3x}$

$$\frac{dy}{dx} = \frac{1}{1+(e^{3x})^2} \cdot 3e^{3x}$$

$$= \frac{3e^{3x}}{1+e^{6x}}$$

3

(c) $y = \sin^{-1} x^4$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(x^4)^2}} \cdot 4x^3$$

$$= \frac{4x^3}{\sqrt{1-x^8}}$$

3

4) (a) $y = 4\tan^2 3x$

$$\frac{dy}{dx} = 8\tan 3x \cdot \sec^2 3x \cdot 3$$

$$= 24 \tan 3x \sec^2 3x$$

3

(b) $y = e^{\cot 2x}$

$$\frac{dy}{dx} = e^{\cot 2x} \cdot -\operatorname{cosec}^2 2x \cdot 2$$

$$= -2e^{\cot 2x} \operatorname{cosec}^2 2x$$

3

5) (a) $f(x) = \cos^2 x \, e^{\tan x}$

$u = \cos^2 x \qquad v = e^{\tan x}$

$u' = 2\cos x \cdot (-\sin x) \qquad v' = e^{\tan x} \cdot \sec^2 x$ ✓

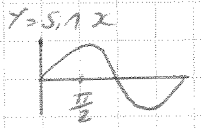
$= -2\cos x \sin x$

$$f'(x) = u'v + uv'$$

$$= -2e^{\tan x}\cos x \sin x + e^{\tan x}$$ ✓ ✓

$$= -e^{\tan x}(2\cos x \sin x - 1)$$

$$= -e^{\tan x}(\sin 2x - 1)$$



$$f'\left(\frac{\pi}{4}\right) = -e^{\tan \frac{\pi}{4}}\left(\sin \frac{\pi}{2} - 1\right)$$ ✓

$$= -e^1 \times (1 - 1)$$

$$= \underline{\underline{0}}$$ ✓

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(b) $g(x) = \dfrac{\tan^{-1} 2x}{1 + 4x^2}$

Quotient rule

$$g'(x) = \frac{\frac{1}{1+(2x)^2} \cdot 2 \cdot (1 + 4x^2) - \tan^{-1}(2x) \cdot 8x}{(1 + 4x^2)^2}$$ ✓ ✓

$$= \frac{\frac{2(1+4x^2)}{1+4x^2} - 8x\tan^{-1}(2x)}{(1+4x^2)^2}$$

$$= \underline{\underline{\frac{2\left(1 - 4x\tan^{-1}(2x)\right)}{(1+4x^2)^2}}}$$ ✓

3

6) $f(x) = \ln(\cos^{-1} 2x)$

$$f'(x) = \frac{1}{\cos^{-1} 2x} \cdot \frac{d}{dx}(\cos^{-1} 2x)$$

$$= \frac{1}{\cos^{-1} 2x} \cdot -\frac{2}{\sqrt{1-(2x)^2}}$$

$$= -\frac{2}{\cos^{-1} 2x \cdot \sqrt{1-4x^2}}$$

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TOTAL = 40 marks