AH Differentiation 1

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1) $f(x) = 2x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{2(x+h)^2 - 2x^2}{h} \quad \checkmark$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2) - 2x^2}{h}$$

$$= \lim_{h\to 0} \frac{2x^2+4xh+2h^2 - 2x^2}{h} \quad \checkmark$$

$$= \lim_{h\to 0} \frac{h(4x+2h)}{h} \quad \checkmark$$

$$= \lim_{h\to 0} (4x+2h)$$

$$= \underline{4x} \quad \checkmark$$

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2) (a) $y = \dfrac{1}{(x^2+3x+5)^2}$

$$= (x^2+3x+5)^{-2} \quad \checkmark$$

$$\frac{dy}{dx} = -2(x^2+3x+5)^{-3} \cdot (2x+3) \quad \checkmark \quad \checkmark \quad \text{(chain rule)}$$

$$= -\frac{2(2x+3)}{(x^2+3x+5)^3}$$

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(b) $y = \sin^2\left(2x - \frac{\pi}{6}\right)$

$$= \left(\sin\left(2x-\tfrac{\pi}{6}\right)\right)^2$$

$$\frac{dy}{dx} = 2\sin\left(2x-\tfrac{\pi}{6}\right) \cdot \cos\left(2x-\tfrac{\pi}{6}\right) \cdot 2 \quad \checkmark \quad \checkmark \quad \checkmark \quad \text{(chain rule)}$$

$$= \underline{4\sin\left(2x-\tfrac{\pi}{6}\right)\cos\left(2x-\tfrac{\pi}{6}\right)} \quad \checkmark$$

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[OR $2\sin 2\left(2x-\frac{\pi}{6}\right)$ ← using $\sin 2A = 2\sin A \cos A$]

3) $f(x) = x^3 \cos 2x$

$f'(x) = 3x^2 \cos 2x + x^3 \cdot (-\sin 2x) \cdot 2$ ✓ ✓ (Product rule)

$= \underline{x^2(3\cos 2x - 2x \sin 2x)}$ ✓ /3

4) (a) $f(x) = \dfrac{x^2}{2x+3}$

$f'(x) = \dfrac{2x(2x+3) - x^2 \cdot 2}{(2x+3)^2}$ ✓ ✓ (Quotient rule)

$= \dfrac{4x^2 + 6x - 2x^2}{(2x+3)^2}$

$= \dfrac{2x^2 + 6x}{(2x+3)^2}$ ✓

$= \dfrac{2x(x+3)}{(2x+3)^2}$ /3

(b) $f(x) = \dfrac{\sin x}{x^2}$

$f'(x) = \dfrac{\cos x \cdot x^2 - \sin x \cdot 2x}{(x^2)^2}$ ✓ (Quotient rule)

$= \dfrac{x(x\cos x - 2\sin x)}{x^4} = \dfrac{x\cos x - 2\sin x}{x^3}$ ✓ ✓ /3

5) $g(x) = \dfrac{\sin x}{1+\cos x}$

$g'(x) = \dfrac{\cos x \cdot (1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$ ✓ ✓ (Quotient rule)

$= \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ ✓ ← (Identity $\cos^2 x + \sin^2 x = 1$) /3

$= \dfrac{\cos x + 1}{(1+\cos x)^2}$ ✓ $= \dfrac{(1+\cos x)}{(1+\cos x)(1+\cos x)} = \dfrac{1}{1+\cos x}$ ✓