DIFFERENTIAL EQUATIONS

Solution of First Order Differential Equations Differential Equations

A differential equation is an equation containing a derivative (or differential quotient).

e.g.
$$y \frac{dy}{dx} + 2xy = x$$
 is a first order differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 4 = \sin x \quad \text{is a } \underline{\text{second order}} \text{ differential equation}$$

To solve differential equations:-

- we find a function y which satisfies the original equation
- we use integration

Differential equations may have a general solution:-

e.g.
$$\frac{dy}{dx} = 2x + 3$$

$$\Rightarrow \int dy = \int (2x + 3) dx$$

$$\Rightarrow y = x^2 + 3x + C$$

or the same differential equation may have a particular solution, if initial conditions are given:-

e.g. Solve
$$\frac{dy}{dx} = 2x + 3$$
, given that y=6 when x=1

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1.4 Applying Calculus Skills to Solve Differential Equations

integrate
$$y = \int (2x+3) dx$$

 $y = x^2 + 3x + c$.
When $x = 1$, $y = 6$ so $6 = 1 + 3 + c$
 $c = 2$
Solution $y = x^2 + 3x + 2$

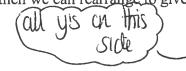
p74 Exercise 6 Questions 1(a), (d), (e), (f), (j), (k)Maths in Action Book 2 2(a), (c), (e) 3, 4, 5, 6

Type 1 - Variables Separable

If the parts of the differential equation involving x and y can be separated from each other to give

$$\frac{dy}{dx} = f(x)g(y)$$

then we can rearrange to give



$$\frac{dy}{g(y)} = f(x)dx$$



and integrate both sides.

Examples

1. Solve
$$(x-1)\frac{dy}{dx} = y$$

Re arrange $\frac{dy}{y} = \frac{dx}{x-1}$

Integrate $\int \frac{dy}{y} = \int \frac{dx}{x-1}$
 $lny = ln[x-1] + c$.

*** To get rid of logs we can let $c = lnk$
 $lny = ln[x-1] + lnk$
 $lny = ln[x-1]$
 $y = k[x-1]$

Aiways answer in

2. Find the general solution of $\frac{dy}{dx} = e^{2x-3y}$ giving y in terms of x.

$$\frac{dy}{dx} = e^{2x}e^{-3y}$$

$$\frac{dy}{dx} = e^{2x}dx$$
Integrate
$$\int e^{3y}dy = \int e^{2x}dx$$

$$\frac{1}{3}e^{3y} = \frac{1}{2}e^{2x}+c$$

$$e^{3y} = \frac{3}{2}e^{2x}+c$$

$$1 \text{ Take logs}$$

$$\ln e^{3y} = \ln \left(\frac{3}{2}e^{2x}+d\right)$$

$$3y = \ln \left(\frac{3}{2}e^{2x}+d\right)$$

$$y = \frac{1}{3}\ln \left(\frac{3}{2}e^{2x}+d\right)$$

3. Find the particular solution of $x^2 \frac{dy}{dx} = 1$, given that y=3 when x=1.

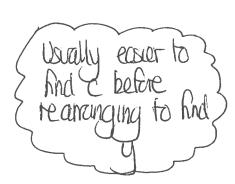
1.4 Applying Calculus Skills to Solve Differential Equations

4. Find the particular solution of $\frac{dy}{dx} = xy - 2x$, given that y=5 when x=0.

Factorise first
$$\frac{dy}{dx} = xy - 2x$$

Factorise first $\frac{dy}{dx} = x(y-2)$
 $\frac{dy}{dx} = xdx$
 $\frac{dy}{y-2} = xdx$
 $\int \frac{dy}{y-2} = \int x dx$
 $\int \frac{dy}{y-2} = \frac{x^2}{x^2} + c$

So
$$\ln |y-2| = \frac{x^2 + \ln 3}{2}$$
 $\ln |y-2| = \frac{x^2}{2} + \ln 3$
 $\frac{y-2}{3} = \frac{x^2}{2} + 2$
 $y = 3e^{\frac{x^2}{2}} + 2$



1.4 Applying Calculus Skills to Solve Differential Equations

5. Find the particular solution of $(1+x^2)\frac{dy}{dx} = 4e^y$, given that y=0 when x=0.

$$\frac{dy}{dey} = \frac{1}{1+x^2} dx$$

Integrate
$$\int \frac{dy}{yey} = \int \frac{1}{1+x^2} dx$$
$$-\frac{1}{1+e^{-y}} = \tan^{-1}x + c$$

When
$$x=0$$
, $y=0$

$$-\frac{1}{4} = tan-0+c$$

$$c=-\frac{1}{4}$$

$$-\frac{1}{4} = y=-tan-x - \frac{1}{4}$$

$$e^{-y} = [-tan-x]$$

$$-y=-tan-x - \frac{1}{4}$$

$$-y=-tan-x - \frac{1}{4}$$

$$y=-tan-x - \frac{1}{4}$$

Tips for success.....

of solving a first order de le only contains dy (not dry) try separating svarables so y's an order side and x's on other.

· integrate + c.

- Find answer must be written as y = x - x- If given initial conditions find a first before rearranging

Maths in Action Book 2 page 74; Exercise 6; Q1e, f, g; Q2c, e; Q4. page 76; Exercise 7; Q1c, d, h; Q2a, b, e. y. page 77; Exercise 8; Q1a, c, e, g, h; Q2, Q3, Q4, Q5, Q6, Q7, Q10.

Type 2 Linear First Order Differential **Equations**

Linear first order differential equations are those that can be written in the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$
 where $P(x)$ and $Q(x)$ are expressions in x.

How to Solve

$$\frac{dy}{dx} + P(x)y = Q(x)$$

We multiply through by
$$e^R$$
 where $R = \int P(x)dx$ What is infinitely of y

This gives

$$e^{R} \frac{dy}{dx} + e^{R} P(x) y = e^{R} Q(x)$$

The left hand side is now an exact derivative.

$$\frac{d}{dx}(e^{R}y) = e^{R}Q(x)$$

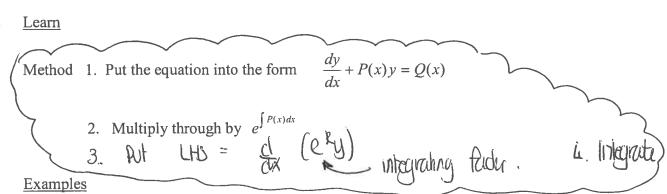
$$= e^{R}\frac{dy}{dx} + ye^{R}\frac{dR}{dx}$$

$$= e^{R}\frac{dy}{dx} + e^{R}P(x).y$$
Since $R = \int P(x)dx$

$$\frac{dR}{dx} = P(x)$$

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Note e^{R} converts the whole of the left side into the derivative of the one function and × X it is called the integrating factor.



Examples

1. Solve
$$\frac{dy}{dx} + \frac{y}{x} = 1$$

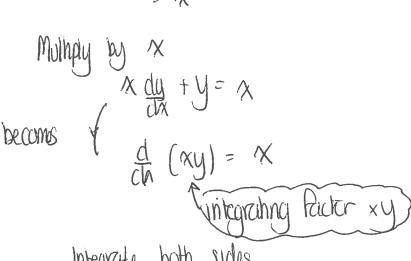
$$\frac{dy}{dx} + \frac{1}{x} = \frac{1}{y} = 1$$
Integrating factor
$$\frac{dy}{dx} + \frac{y}{x} = 1$$

$$\frac{dy}{dx} + \frac{1}{x} = \frac{1}{y} = 1$$
Integrating factor
$$\frac{dy}{dx} + \frac{y}{x} = 1$$

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$$\frac{dy}{dx} + \frac{1}{x} = 1$$
Integrating factor
$$\frac{dy}{dx} + \frac{1}{x} = 1$$

$$\frac{dy}{dx$$



Integrate both sides
$$xy = \int x \, dx$$

$$xy = \frac{x}{2} + c$$

$$y = \frac{x}{2} + c$$

VERY IMPORTANT

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1.4 Applying Calculus Skills to Solve Differential Equations

2. Solve
$$x \frac{dy}{dx} - (x-2)y = \frac{1}{x}$$

* Make plain dy first *

$$\frac{dy}{dx} - \frac{x-2}{x} y = \frac{1}{x^2}$$

Unicograting factor

$$e \int_{-\frac{x-2}{x}}^{-\frac{x-2}{x}} dx$$

$$= e \int_{-\frac{x-2}{x}}^{-\frac{x-2}{x}} dx$$

$$= e^{2\ln x - x}$$

$$= e^{2\ln x - x}$$

$$= e^{2\ln x}$$

$$=$$

 $\frac{x^2}{e^x} \frac{dy}{dx} - \frac{x^2(x-2)}{e^x x} y = \frac{1}{e^x}$ becomes f $\frac{d}{dx}\left(\frac{x^2}{2x}y\right) = \frac{1}{2x}$

T integreting factor xy

$$\frac{x^2}{e^x}y = \int \frac{1}{e^x} dx$$

$$\frac{x^2}{e^x}y = -e^{-x} + c$$

$$\frac{x^2}{e^x}y = c - \frac{1}{e^x}$$

$$y = c - \frac{1}{e^x}$$

$$y = c - \frac{1}{e^x}$$

DILL EXI

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linear

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3. Find the particular solution of

Make plain cly

$$\frac{dy}{dx} + y = (x+1)^{2}e^{x} \text{ given that } x = 1 \text{ when } y = 0$$

Make plain cly
$$\frac{dy}{dx} + \frac{1}{x}(x+1) y = \frac{x+1}{x} e^{x} \qquad (x)$$

Integrating factor $e^{\int x(x+1) dx}$

$$\frac{1}{x} + \frac{1}{x} +$$

Tips for success.....

dy + - y = - => linear hype.

make size plain dy

Rnd integraling factor es (bit in front of y) dx

multiply by integraling factor so LHS = d/ integraling

Integral

Tind c if appropriete of write answer as y =

Applications of First Order Differential Equations

A large number of situations where *rates of change* are involved can be modelled using differential equations.

Common applications include :-

- Populations: growth and decay
- Law of cooling: where the rate at which an object cools is proportional to the temperature difference between it and its surroundings.

Given the situation we must first construct a model (in the form of a differential equation) and the use the usual procedures to solve.

Note

1. Remember
$$\frac{dy}{dx}$$
 means rate of change of y with x .

2. Rate of change of P with t varies directly as t means

$$\frac{dP}{dt}$$
 at ie $\frac{dP}{clt}$ = kt (k = constant)

3. Rate of change of Q with r varies inversely as r means

$$\frac{d\theta}{dr} \propto \frac{1}{r}$$
 ie $\frac{d\theta}{dr} = \frac{k}{r}$

4. For motion of a body in a straight line

$$V = \frac{dS}{dt}$$
 $Cl = \frac{dV}{dt}$

Example 1

The rate at which a radioactive material decays at any instant is proportional to the mass remaining at any instant. Given that there are x grams present after t days, the differential equation in x is modelled by:-

$$\frac{dx}{dt} = -kx$$

The half-life (i.e. the time taken for half the mass to decay) of the radioactive material is 25 days.

Find the time taken for 100g of the material to decay to 20g.

Integrate
$$\frac{dx}{dt} = -kx$$

$$\frac{dx}{dt} = -kdt$$

$$\ln x = -kt + c$$
When $t=0$, $x=100 \Rightarrow \ln 00 = c$

$$t=25 \quad x=50 \Rightarrow \ln 50 = -25k + \ln 00$$

$$\ln \frac{1}{2} = -25k$$

$$k = -\frac{1}{25} \ln \frac{1}{2}$$
So
$$\ln x = \left(\frac{1}{25} \ln \frac{1}{2}\right) + \ln 100$$

$$\ln \frac{1}{5} = \left(\frac{1}{25} \ln \frac{1}{2}\right) + \ln 100$$

$$\ln \frac{1}{5} = \left(\frac{1}{25} \ln \frac{1}{2}\right) + \ln 100$$

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Example 2

An infectious disease spreads at a rate which is proportional to the product of the number infected and the number uninfected.

Initially one half of the population is infected and the rate of spread is found to be $\frac{1}{48}$ of the population per day.

Calculate the proportion of the population which will be infected after 12 days.

Let
$$P$$
 = number of population (constraint) X = number of people inflocted X = number of people inflocted X = number uninflected X = X (X = X (X = X (X = X (X = X) X = X (X = X (X = X (X = X) X = X (X = X (X = X) X = X (X = X (X = X) X = X = X + X = X

1.4 Applying Calculus Skills to Solve Differential Equations

Unix -
$$\ln (P-x) = \frac{1}{12}t + c$$

Unix - $\ln (P-x) = \frac{1}{12}t + c$

Unix - $\ln (P-x) = \frac{1}{12}t + c$

Unix - $\ln (\frac{1}{2}P) = c$

So $\ln \frac{x}{P-x} = \frac{1}{12}t$
 $\frac{x}{P-x} = e^{\frac{1}{12}t}$

When $t=12$
 $\frac{x}{P-x} = e$
 $\frac{x}{P-$

Maths in Action Book 2 page 81; Exercise 9A; Q1, 3, 4, 5, 6, 7, 8, 10. page 83; Exercise 9B; Q1, 2, 3, 5, 7, 12.

Tips for success.....

- Remember to define any letters used if
- vanes directly => letter goes on top vanus inversely => letter goes on bottom
 - · Some as before

Solving Quadratic Equations when the Discriminant is Negative

We define
$$\sqrt{-1} = i$$

eg
$$\sqrt{-4} = \sqrt{4} \times \sqrt{-1}$$
$$= 2i$$
$$\sqrt{-81} = \sqrt{81} \times \sqrt{-1}$$
$$= 9i$$

These are called imaginary numbers.

Complex numbers have a real part and an imaginary part. eg 2 + 5i , 6 - 8i

The set of complex numbers is denoted by C

All quadratic equations can be solved if we use complex numbers.

Example

Solve
$$x^2 + 2x + 5 = 0$$

$$\alpha = 1 \qquad b = 2 \qquad C = 5$$

$$A = -b = 1 \qquad b^2 - 4aC$$

$$2a$$

$$= -2 = 1 \qquad 14 - 4x + 15$$

$$2$$

$$= -2 = 4i$$

$$2$$

$$A = -1 = 2i$$

Second Order Differential Equations

"Simple" Type:- i.e.
$$\frac{d^2y}{dx^2} = f(x)$$

Solve these by integrating twice.

e.g.
$$\frac{d^2 y}{dx^2} = x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^3}{3} + c$$

$$\Rightarrow y = \frac{x^4}{12} + cx + d$$

Note that like all 2nd order differential equations the solution MUST have 2 arbitrary constants.

Second Order Linear Differential Equations

In general, these take the form :- $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = Q(x)$

dy must be there at 0

but dy or y could be missing

(a) Homogeneous 2nd Order Linear Differential Equations

Initially we will concentrate on the case when Q(x)=0.

This is known as a homogeneous 2nd order differential equation:-

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$
... equation (1)

The differential equation $b \frac{dy}{dx} + cy = 0$ has a solution of the form $y = Ae^{mx}$

$$y = Ae^{mx}$$

$$\Rightarrow \frac{dy}{dx} = Ame^{mx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = Am^2e^{mx}$$

$$\frac{dy}{dx} = -\frac{cy}{dx}$$

$$\frac{dy}{dy} = -\frac{c}{b}\frac{dx}{dx}$$

$$\frac{dy}{dx} = -\frac{c}{b}\frac{dx}{dx}$$

$$\frac{dx}{dx} = -\frac{c}{b}\frac{dx}{dx}$$

$$\frac{dx}{dx} = -\frac{c}{b}\frac{dx}{dx}$$

$$\frac{d$$

1.4 Applying Calculus Skills to Solve Differential Equations

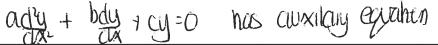
Substituting $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ into the original equation (1), gives:

$$aAm^2e^{mx} + bAme^{mx} + cAe^{mx} = 0$$
, dividing all by Ae^{mx}
(assuming $Ae^{mx} \neq 0$)

we obtain
$$am^2 + bm + c = 0$$

This is known as the **Auxiliary Equation** and its solutions for m provide 2 values of m which make $y = Ae^{mx}$ a valid solution for our original equation (1).

Distinguishing between the 2 roots of m using m_1 and m_2 give us 2 solutions for equation 1.







The roots of the Auxiliary Equation may take 3 forms:-

2 real distinct roots $(b^2 - 4ac > 0)$ (i)

 $m = m_1$ and m_2 and the general solution of equation (1) is:-

$$y = Ae^{m_1x} + Be^{m_2x}$$

(ii) Equal roots $(b^2 - 4ac = 0)$

 $m = m_1$ (twice) and the general solution of equation (1) is:-

$$y = Ae^{m_1 x} + Bxe^{m_1 x}$$
 = $(A + Bx)e^{m_1 x}$ see AH3 page 119

2 complex roots $(b^2 - 4ac < 0)$ (iii)

> $\emph{m}=\emph{m}_1$ and \emph{m}_2 , but \emph{m}_1 and \emph{m}_2 are of the form $\emph{p}\pm\emph{q}\emph{i}$ and the general solution of equation (1) is :-

$$y = Ae^{px}\cos qx + Be^{px}\sin qx$$

see AH3 page121

Example 1

Find the general solution of:-

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

Availary equation
$$m^2 + m - 6 = 0$$

 $(m+3)(m-2) = 0$
 $m = -3$ or $m = 2$ (2 distinct rook)

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Example 2

Solve
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$$

Close the factorise \Rightarrow use quadratic formula

 $M = -b \pm \sqrt{b^2 - 4ac}$
 $a = 1, b = 2, c = 5$
 $2ac$
 $M = -2 \pm \sqrt{2^2 - 20}$
 $M = -2 \pm 4c$
 $m = -1 \pm 2c$

General Solution

 $y = Ae^{-x} \cos 2x + Be^{-x} \sin 2x$

Maths in Action Book 3 page 119 Exercise 3 Question 1(a) (d) (e) page 120 Exercise 4 Question 1(a) (d) (e) page 122 Exercise 5 Question 1(a) (d) (e)

1.4 Applying Calculus Skills to Solve Differential Equations

0

Given y(0) = 2 and y'(0) = 9, solve the differential equation:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0.$$

auxilary equation

$$m^2-6m+q=0$$

 $(m-3)(m-3)=0$
 $m=3$ equal roots.

General equation $y = Ae^{3x} + Bx e^{3x}$ The value so this term is different from Ae^{3x}

$$y(0) = 2$$

 $ie^{7} x = 0, y = 2$ $so 2 = A$

$$y'(0) = 9$$
 $i\bar{e} \times = 0$ $dy = 9$
 $i\bar{e} \times = 0$ $dy = 9$
 $dx = 3Ae^{3x} + Bx \cdot 3e^{3x} + Be^{3x}$
 $i\bar{e} \times = 0$
 $i\bar{e} \times = 0$

When
$$X=0$$
 dy = 9 So $9=3A+8$
 $9=6+8$
 $8=3$
Solution $y=2e^{3x}+3xe^{3x}$

Maths in Action Book 3 page 119 Exercise 3 Question 2(a) (d) page 120 Exercise 4 Question 2(a) (d) page 122 Exercise 5A Question 1(a) (d) page 122 Exercise 5B

General Solution of a Non-Homogeneous Equation

We are now going to consider solving second order differential equations of the form

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$
(1)

First we find the general solution to the corresponding homogeneous equation

$$a\frac{d^{2}y}{dx^{2}} + b\frac{dy}{dx} + cy = 0$$
(2)

The solution is the form

$$y = Ag(x) + Bh(x)$$

This is called the <u>complementary function</u> C. F.

Second we find a particular solution (<u>particular integral</u>) P.I. of the non-homogeneous equation (1)

$$y = P(x)$$

The general solution we want is

$$y = Ag(x) + Bh(x) + P(x)$$

or
$$y = C.F + P.I.$$

<u>How to find the P.I.</u> Look at f(x) the right hand side of (1) and choose the correct P.I. from the list.

f(x)	Correct form for P.I.
e^{px}	$y = C e^{px}$ (p not a root of the auxiliary equation) $y = Cx e^{px}$ (p a non repeated root of the auxiliary equation) $y = C x^2 e^{px}$ (p a repeated root of the auxiliary equation)
Hcospx + Ksinpx (or either of these by itself)	y = Ccospx + Dsinpx but if this is already the form of the C.F. then use $y = x(Ccospx + Dsinpx)$
linear function px + q	y = Cx + D (m = 0 not a root of the auxiliary equation) y = x(Cx + D) (m = 0 a non-repeated root)
quadratic function $rx^2 + px + q$	$y = Cx^2 + Dx + E$ (m=0 not a root of the auxiliary equation) $y = x(Cx^2 + Dx + E)$ (m = 0 a non repeated root)

<u>Finally</u> substitute your P.I. as chosen above in the original differential equation to find constants C and D.

Example

Solve
$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4x + 4$$

$$\text{auxility equation}$$

$$m^2 + 3m + 2 = 0$$

$$(m+2)(m+1) = 0$$

$$m = -2 \text{ or } m = -1$$

$$\text{C.F. } y = Ae^{-2x} + Be^{-x}$$
P.T. Consider RHS= $4x + 6$

$$\text{Try } y = Cx + 1$$

$$\frac{dy}{dx} = C$$

$$\frac{dy}{dx} = 0$$
Substitute with $\frac{dy}{dx} + 3\frac{dy}{dx} + 2y = 4x + 4$

$$0 + 3C + 2Cx + 21 = 4x + 4$$
Equation Goefficients
$$\text{constant } 3C + 21 - 4 - - 0$$

$$x \text{ term. } 2C = 4$$

$$C = 2$$

$$1 + 0 + 21 - 4$$

$$21 - 2$$

$$1 - 1$$

$$21 - 2$$

$$1 - 1$$

$$21 - 2$$

$$1 - 1$$

$$22 - 4$$

$$22 - 4$$

$$1 - 2 - 4$$

$$23 - 4$$

$$24 - 4$$

$$24 - 4$$

$$25 - 4$$

$$27 - 4$$

$$27 - 4$$

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Example

solve
$$\frac{d^{3}y}{dx^{2}} + 6\frac{dy}{dx} + 10y = 30\sin 2x$$

$$2a + 6\frac{4}{36} + 10 = 0$$

$$m = -\frac{b}{3} + \frac{1}{36} + \frac{10}{40}$$

$$m = -\frac{6}{4} + \frac{2}{2}i$$

$$m = -3 \pm i$$

$$CF \quad y = Ae^{3x}\cos x + Be^{-3x}\sin x$$

$$PT \quad hook \quad (if \quad RHS = 30\sin 2x)$$

$$Try \quad y = C\cos 2x + 1\sin 2x$$

$$\frac{NB}{M} \quad must \quad have \quad both \quad trig \quad bits.$$

$$\frac{dy}{dx} = -2\cos 2x + 2\cos 2x + 2\cos 2x$$

$$\frac{dy}{dx} = -4\cos 2x - 4\cos 2x$$

$$\frac{dy}{dx} = -4\cos 2x - 4\cos 2x$$

$$\frac{dy}{dx} = -4\cos 2x + 1\cos 2x$$

$$\frac{dy}{dx} = -3\cos 2x + 4\cos 2x + 1\cos 2x$$

$$\frac{dy}{dx} = -3\cos 2x + 4\cos 2x + 1\cos 2x$$

$$\frac{dy}{dx} = -3\cos 2x + 4\cos 2x + 1\cos 2x$$

$$\frac{dy}{dx} = -3\cos 2x + 3\cos 2x + 3\cos 2x$$

$$\frac{dy}{dx} = -3\cos 2x + 3\cos 2x + 3\cos 2x + 3\cos 2x$$

$$\frac{dy}{dx} = -3\cos 2x + 3\cos 2x + 3\cos 2x + 3\cos 2x$$

$$\frac{dy}{dx} = -3\cos 2x + 3\cos 2x + 3$$

Equate
$$CCefficients$$

$$-4C+120+10C=0$$

$$120+6C=0$$

$$C=-2D$$

1.4 Applying Calculus Skills to Solve Differential Equations

(Sin 2x)
$$-40 - 12C + 10D = 30$$

 $-4D + 24D + 10D = 30$
 $30D = 30$
 $D = 1$
 $C = -2$
P.I $y = -2\cos 2x + \sin 2x$
General Solution $y = CF + PI$
 $y = Ae^{-3x}\cos x + Be^{-3x}\sin x - 2\cos 2x + \sin 2x$.

Example

Find the particular solution to the equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x$$

given that x = 0, y = 1 and $\frac{dy}{dx} = 2$

Quailogy equality
$$m = 3m + 2 = 0$$

 $(m-2)(m-1) = 0$
 $m = 2$ or $m = 1$
 CF $y = Ae^{2x} + Be^{2x}$
PI Try $y = Cxe^{2x}$
 Cxe^{2x}
 Cxe^{2x}

1.4 Applying Calculus Skills to Solve Differential Equations

Subshible into
$$\frac{dy}{dx}$$
 - $\frac{3dy}{dx}$ + $\frac{2y}{e^x}$ = e^x
 $\frac{(2x)^x}{(2x)^x}$ + $\frac{2(e^x)^x}{(2x)^x}$ - $\frac{3((2x)^x)^x}{(2x)^x}$ + $\frac{2((2x)^x)^x}{(2x)^x}$ = e^x
 $\frac{-(e^x)^x}{(2x)^x}$ - $\frac{2(2x)^x}{(2x)^x}$ = e^x
 $\frac{-(e^x)^x}{(2x)^x}$ = e^x
 $\frac{(2x)^x}{(2x)^x}$ + $\frac{2((2x)^x)^x}{(2x)^x}$ = e^x
 $\frac{-(e^x)^x}{(2x)^x}$ = e^x e^x = e^x e^x = e^x

Now
$$x=0$$
, $y=1 \Rightarrow 1 = A+B = 0$
From (x) $dy = 2Ae^{2x} + Be^{x} - (xe^{x} + e^{x})$
Now $x=0$, $dy = 2$ so $2 = 2A+B-1$
 $2A+B=3 = 0$
2 -0 gives $A=2$
 $\Rightarrow B=-1$
Particular solution $y=2e^{2x}-e^{x}-xe^{x}$

Maths in Action Book 3 Page 126 Exercise 7A Questions 1(a), (b), (e), (f), (g), (j)
Page 127 Exercise 7B Questions !(a), (e),