

3.1

Constructing and solving equations

Key words
solve
construct

✚ Solve problems by constructing and solving equations

When **solving** equations involving brackets it is essential that you simplify each side of the equation as much as possible before solving.

When solving equations we must always do the same to both sides of the equation to ensure it remains balanced.

Often we can **construct** equations to help solve problems that occur in real life.

Example

James and Adrian are the same age. James calculates the amount of pocket money he is given by multiplying his age by 5 and adding 50p.

Adrian works out his pocket money by adding 25p to his age and multiplying by four.

- What age will they be when they receive the same amount of pocket money?
- Comment on your answer.

a) The amount of pocket money James receives is: $5x + 50$
 The amount of pocket money Adrian receives is: $4(x + 25)$
 $5x + 50 = 4(x + 25)$
 $5x + 50 = 4x + 100$
 $x + 50 = 100$
 $x = 50$

b) They are unlikely to still be given pocket money when they are 50!

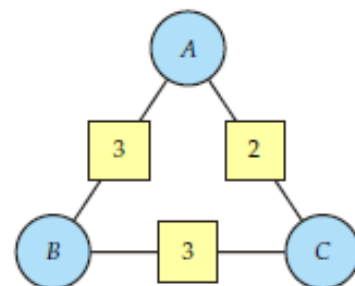
Let x represent James and Adrian's age when they receive the same amount of pocket money.

When they both receive the same amount the two expressions are equal.

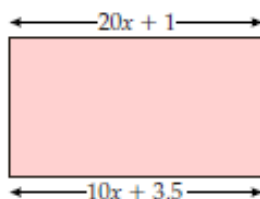
Solve the equation to find the value of x .

Exercise 3.1

- In an arithmagon, the number in a square is the sum of the numbers in the two circles on either side of it.
 Let the value in circle A be represented by x .
 Write down algebraic expressions for the values in circles B and C and hence find the values of A , B and C .



- The area of this rectangle is 18 cm^2 .

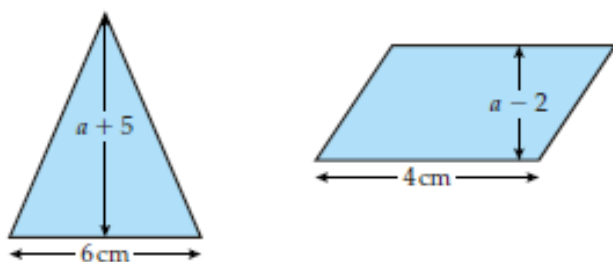


Calculate the value of x and use it to find the dimensions of the rectangle.

- 3 In triangle ABC $\angle A$ is half of $\angle B$, which is a third of $\angle C$. Find all the angles in the triangle.

Let x represent the size of $\angle C$.

- 4 These two shapes have the same area.



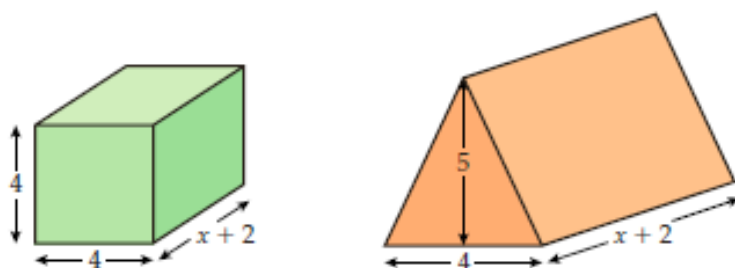
Find the area of the shapes.

- 5 The cost of a CD increases by 10%. The new cost is now £15.95. Form and solve an equation to find the original cost of the CD.

Look back to Lesson 2.6.

- 6 The number of pupils at Langford High School is 200 short of being double the number of pupils at Court Senior School.
- If the number of pupils at Court Senior School is m how many pupils are there at Langford High School?
 - If there are 500 pupils at Langford High, how many are at Court Senior?
- 7 Both Winifred and Winston collect model helicopters. Between them they have 200. If Winifred sold Winston 50 then Winston would have four times as many as Winifred. How many does Winifred have?

- 8 These two solid blocks have the same volume.



Find the volumes of the shapes.

3.2

Solving more complex equations

Key words
solve
trial and improvement

Use either algebraic methods or trial and improvement methods to solve equations

We can **solve** equations involving powers of x using **trial and improvement** methods.

For example, find the value of x when $x^2 + x = 10$ to one decimal place.

Choose values of x and substitute them into the equation.

$$x = 2 \quad x^2 + x = 2^2 + 2 = 6 \quad \text{too small}$$

$$x = 3 \quad x^2 + x = 3^2 + 3 = 12 \quad \text{too large}$$

The value of x lies between 2 and 3.

$$x = 2.5 \quad x^2 + x = 2.5^2 + 2.5 = 8.75 \quad \text{too small}$$

$$x = 2.7 \quad x^2 + x = 2.7^2 + 2.7 = 9.99 \quad \text{too small}$$

$$x = 2.8 \quad x^2 + x = 2.8^2 + 2.8 = 10.64 \quad \text{too large}$$

The value of x lies between 2.7 and 2.8.

$$x = 2.7$$

Since the value of $x^2 + x$ is closer to 10 when $x = 2.7$, we have found the solution to one decimal place.

Example

Solve the following equations:

a) $5y^2 + 9 = 11$

b) $3b + b^3 = 100$

We can solve this equation algebraically since there is only one term containing an unknown.

a) $5y^2 + 9 = 11$

$$5y^2 = 2 \quad (\text{Subtract 9 from both sides})$$

$$y^2 = 0.4 \quad (\text{Divide both sides by 5})$$

$$y = \pm 0.63 \quad (\text{Square root both sides})$$

Remember, when we take the square root of a number there are always two solutions.

b) $3b + b^3 = 100$

$$b = 5 \quad 3 \times 5 + 5^3 = 15 + 125 = 140 \quad \text{too big}$$

$$b = 4 \quad 3 \times 4 + 4^3 = 12 + 64 = 76 \quad \text{too small}$$

The value of b must lie between 4 and 5.

$$b = 4.5 \quad 3 \times 4.5 + 4.5^3 = 13.5 + 91.125 = 104.625 \quad \text{too big}$$

$$b = 4.4 \quad 3 \times 4.4 + 4.4^3 = 13.2 + 85.184 = 98.384 \quad \text{too small}$$

98.384 is closer to 100 than 104.625 so $b = 4.4$

(to one decimal place)

There are two terms containing an unknown so we must use trial and improvement techniques.



Exercise 3.2

- 1 Use algebraic methods to solve the following equations, giving your answer to two decimal places:

a) $x^2 + 4 = 20$

b) $2t^2 = 70$

c) $b^2 - 5 = 80$

d) $\frac{m^2}{4} = 1$

e) $10 = 2z^2 + 7$

f) $\frac{2w^2}{5} = 41$

- 2 Use trial and improvement techniques to solve the following equations to the level of accuracy given. You may use ICT if you wish.

a) $r^3 + r = 20$ (2 significant figures)

b) $b^2 - 4b = 100$ (2 decimal places)

c) $0.5 = m^2 + m$ (3 significant figures)

d) $q^5 - 20 + q = 0$ (1 decimal place)

- 3 Decide whether you would use trial and improvement techniques (T) or algebraic techniques (A) to solve each of the following equations.

a) $4x^2 = 25$

b) $g^3 + g = 300$

c) $8v - v^2 = 1$

d) $12 - 3y^2 = -55$

e) $x^2 - 5 = 20$

f) $\frac{5}{f^2} = 4$

g) $30 = 5h^3 + h^2$

If you are unsure in which order to carry out the operations draw a function machine.

- 4 Solve each of the equations in Q3, giving your answers to 1 decimal place.

- 5 To work out the area of material required for a pair of curtains, a seamstress squares the height of the window and adds 0.5 m.

a) Write down an algebraic expression for the amount of material required when a window has height y .

b) Write down and solve an equation to calculate the height of a window when the area of material required for curtains is 2.75 m^2 .

- 6 A cuboid has equal width and length, $w \text{ cm}$, and height $(w + 1) \text{ cm}$.

a) Write down and simplify an expression for the volume of the cuboid.

b) If the cuboid has volume 1000 cm^3 , form and solve an equation to find the value of w to one decimal place.

c) Sketch a diagram of the cuboid, marking on all the known lengths.

- 7 In a factory cubic cartons are made to hold orange juice.

The cartons must hold 1 litre of orange juice.

Write down as many possible dimensions as you can for the cartons.

Investigate the surface area of the different cartons you have found.

What are the dimensions of the carton whose surface area is smallest?

1 litre = 1000 ml
1 ml = 1 cm³

Use whole numbers only.

3.3

Forming and solving equations

Key words
solve
equation
trial and improvement

Form and solve equations

We can **solve equations** in one of two ways:

Algebraically: by carrying out inverse operations to find the solution.

Trial and improvement: by choosing different values for the unknown and substituting them into the equation.

It is important that we choose the most appropriate technique for solving each equation. Although both methods are acceptable, trial and improvement techniques can be time-consuming.

Example

Jeanette sits two mathematics examinations: paper 1 and paper 2. Her mean mark is 63%. She did better in paper 1 than in paper 2 in which she scored 14% less.

What were her marks in each paper?

Paper 1 = x	Choose a letter to represent Jeanette mark in Paper 1, in this case x . Since she scored 14 less in paper 2 we can express her mark in terms of x .
Paper 2 = $(x - 14)$	
Form an equation:	To find the mean score, add both scores together and divide by 2.
$\frac{(x - 14) + x}{2} = 63$	
$\frac{2x - 14}{2} = 63$	Simplify the equation as much as possible.
Solve the equation:	Having found the value of x , we can calculate the marks Jeanette received on both papers.
$2x - 14 = 126$ (Multiply both sides by 2)	
$2x = 140$ (Add 14 to both sides)	
$x = 70$ (Divide both sides by 2)	
Paper 1 = 70%	
Paper 2 = $70 - 14 = 56$	



Exercise 3.3

For each question in this exercise:

- a) form an equation b) decide how to solve it c) solve the equation.

Give your answers to one decimal place where appropriate.

- 1 A jewellery box has equal length and height but the width is 2 cm less than the length. If the volume of the jewellery box is 1500 cm^3 , find the dimensions of the box.

- 2 Two consecutive odd numbers when squared and added together give a total of 394. Calculate the value of the two numbers.
- 3 Matilda thinks of a number, squares it, divides the answer by 100, and gets 6.25. What number was she thinking of?
- 4 A farmer has 100 m of fence. He wishes to use it to fence off a paddock in which to keep horses.
The width of the field must be 2 m more than the length.
- What are the dimensions of the field?
 - What is the area of the field?
- 5 A triangle's height is 3 cm longer than the length of the base. If the area of the triangle is 80 cm^2 , what are the triangle's dimensions?
- 6 Mr and Mrs Murphy have three children: Milly, Molly and Micky.
Their average height is 68 cm.
Milly is 8 cm shorter than Molly and Mickey is 10 cm taller than Milly.
How tall are Milly, Molly and Mickey?
- 7 The product of three consecutive multiples of 6 is 25920. Find the three numbers.
- 8 In the Summer Examination Alex sits 10 papers. His mean mark for the first five papers is 65. For all 10 papers his mean mark is 72.
What is his mean mark for the last five papers?

Investigation

- 9 A rectangular area in a car park is going to be fenced off for bicycles. 20 metres of fencing has been provided and the area will be against a wall as shown:
- Copy and label the diagram, giving the lengths of fence in terms of r .
 - If the area must be 40 m^2 , what are the dimensions of the space?
 - Can you find more than one answer to part b)?
 - What is the largest area that could be fenced off using 20 metres of fence?

