CfE AH Maths Homework 7
Second Order Differential Equations

Find the general solution of the following differential equations.

1. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 10e^{2x}$

2. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4(x+1)$

3. $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 25y = 26\cos 3x$

4. $4\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 9y = 7e^{-2x}$

5. $\frac{d^2y}{dx^2} + 4y = -4x^2 + 2$

Find the general solution and particular solutions for the following differential equations

6. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = \sin x$ given $y = 0$, $\frac{dy}{dx} = 1$ when $x = 0$.

7. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 5e^{3x}$ given $y = 1$, $\frac{dy}{dx} = -6$ when $x = 0$.

Answers

① $y = Ae^{-3x} + Be^{x} + 2e^{2x}$

② $y = Ae^{-2x} + Be^{-x} + 2x - 2$

③ $y = Ae^{-4x}\cos 3x + Be^{-4x}\sin 3x + \frac{1}{2}\cos 3x + \frac{3}{4}\sin 3x$

④ $y = Ae^{-\frac{3}{2}x} + Bxe^{-\frac{3}{2}x} + 7e^{-2x}$

⑤ $y = A\cos 2x + B\sin 2x - x^2 + 1$

⑥ $y = Ae^{-x}\cos x + Be^{-x}\sin x - \frac{2}{5}\cos x + \frac{1}{5}\sin x$

$y = \frac{2}{5}e^{-x}\cos x - \frac{3}{5}e^{-x}\sin x - \frac{2}{5}\cos x + \frac{1}{5}\sin x$

⑦ $y = Ae^{3x} + Be^{-2x} + xe^{3x}$

$y = -e^{3x} + 2e^{-2x} + xe^{3x}$