CfE AH Maths Homework 5

1. Find $\int 2x \ln(x+1)\,dx$

2. Solve the differential equation

$$\frac{dy}{dx} = \frac{y^2-1}{2\tan x} \quad \text{given that } y=3 \text{ when } x=\frac{\pi}{2}$$

3. Use integration by parts to evaluate $\int_0^1 \ln(1+x)\,dx$

4. Solve the differential equation

$$(x+2)\frac{dy}{dx} = (2x^2+4x+1)(y-3)$$

given that when $x=0$, $y=7$

5. The volume $v(t)$ of a cell at time t changes according to the law

$$\frac{dv}{dt} = v(10-v) \quad \text{for } 0<v<10$$

Show that $\frac{1}{10}\ln v - \frac{1}{10}\ln(10-v) = t+c$

for some constant c.

Given that $v(0)=5$ show that

$$v(t) = \frac{10e^{10t}}{1+e^{10t}}$$

Obtain the limiting value of $v(t)$ as $t \to \infty$