CfE AH Maths Homework (17)

(1) Find the point of intersection of the line

$$\frac{x-3}{4} = \frac{y-2}{-1} = \frac{z+1}{2}$$

and the plane with equation $2x + y - z = 4$. **4**

(2) Given the matrix $A = \begin{pmatrix} t+4 & 3t \\ 3 & 5 \end{pmatrix}$.

(*a*) Find A^{-1} in terms of t when A is non-singular. **3**

(*b*) Write down the value of t such that A is singular. **1**

(*c*) Given that the transpose of A is $\begin{pmatrix} 6 & 3 \\ 6 & 5 \end{pmatrix}$, find t. **1**

(3) (*a*) Find an equation of the plane π_1 through the points $A(1, 1, 1)$, $B(2, -1, 1)$ and $C(0, 3, 3)$. **3**

(*b*) The plane π_2 has equation $x + 3y - z = 2$.
Given that the point $(0, a, b)$ lies on both the planes π_1 and π_2, find the values of a and b. Hence find an equation of the line of intersection of the planes π_1 and π_2. **4**

(*c*) Find the size of the acute angle between the planes π_1 and π_2. **3**

(4) The matrix A is such that $A^2 = 4A - 3I$ where I is the corresponding identity matrix. Find integers p and q such that

$$A^4 = pA + qI.$$ **4**

(5) (*a*) Find an equation of the plane π_1 containing the points $A(1, 0, 3)$, $B(0, 2, -1)$ and $C(1, 1, 0)$.
Calculate the size of the acute angle between π_1 and the plane π_2 with equation $x + y - z = 0$. **4**

(*b*) Find the point of intersection of plane π_2 and the line **3**

$$\frac{x-11}{4} = \frac{y-15}{5} = \frac{z-12}{2}.$$ **3**