Queshuns from AH Paper 2003.

A1.   (a)   Given $f(x) = x(1 + x)^{10}$, obtain $f'(x)$ and simplify your answer.   3

       (b)   Given $y = 3^x$, use logarithmic differentiation to obtain $\dfrac{dy}{dx}$ in terms of x.   3

A2.   Given that $u_k = 11 - 2k$, $(k \geq 1)$, obtain a formula for $S_n = \displaystyle\sum_{k=1}^{n} u_k$.   3

       Find the values of n for which $S_n = 21$.   2

A3.   The equation $y^3 + 3xy = 3x^2 - 5$ defines a curve passing through the point A (2, 1). Obtain an equation for the tangent to the curve at A.   4

A4.   Identify the locus in the complex plane given by $|z + i| = 2$.   3

A5.   Use the substitution $x = 1 + \sin\theta$ to evaluate $\displaystyle\int_0^{\pi/2} \frac{\cos\theta}{(1 + \sin\theta)^3} \, d\theta$.   5
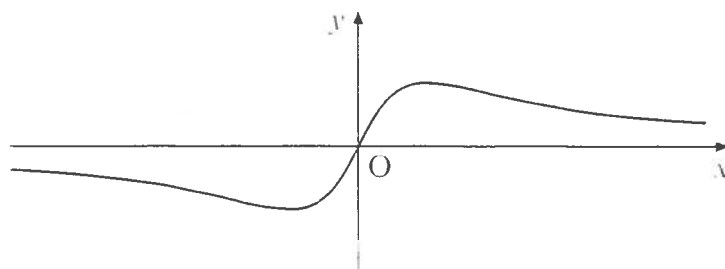
A6.   Use elementary row operations to reduce the following system of equations to upper triangular form

$$
\begin{aligned}
x &+ y + 3z = 1 \\
3x &+ ay + z = 1 \\
x &+ y + z = -1.
\end{aligned}
$$

    2

       Hence express x, y and z in terms of the parameter a.   2

       Explain what happens when $a = 3$.   2

A7.

       The diagram shows the shape of the graph of $y = \dfrac{x}{1 + x^2}$. Obtain the stationary points of the graph.   4

       Sketch the graph of $y = \left| \dfrac{x}{1 + x^2} \right|$ and identify its three critical points.   3

(PTO)

A8. Given that $p(n) = n^2 + n$, where n is a positive integer, consider the statements:

    A $p(n)$ is always even

    B $p(n)$ is always a multiple of 3.

For each statement, prove it if it is true or, otherwise, disprove it.

    4

A9. Given that $w = \cos\theta + i\sin\theta$, show that $\frac{1}{w} = \cos\theta - i\sin\theta$.

    1

Use de Moivre's theorem to prove $w^k + w^{-k} = 2\cos k\theta$, where k is a natural number.

    3

Expand $(w + w^{-1})^4$ by the binomial theorem and hence show that

$$\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}.$$

    5