

CPE AH Maths Homework (14)

Questions from 2002 AH Paper.

- A1.** Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} x + y + 3z &= 2 \\ 2x + y + z &= 2 \\ 3x + 2y + 5z &= 5. \end{aligned}$$

- A2.** Verify that i is a solution of $z^4 + 4z^3 + 3z^2 + 4z + 2 = 0$.
Hence find all the solutions.

- A3.** A curve is defined by the parametric equations

$$x = t^2 + t - 1, \quad y = 2t^2 - t + 2$$

for all t . Show that the point $A(-1, 5)$ lies on the curve and obtain an equation of the tangent to the curve at the point A .

- A4.** (a) Given that $f(x) = \sqrt{x}e^{-x}$, $x \geq 0$, obtain and simplify $f'(x)$.

- (b) Given $y = (x+1)^2(x+2)^{-4}$ and $x > 0$, use logarithmic differentiation to show that $\frac{dy}{dx}$ can be expressed in the form $\left(\frac{a}{x+1} + \frac{b}{x+2}\right)y$,
stating the values of the constants a and b .

- A5.** Use integration by parts to evaluate $\int_0^1 \ln(1+x) dx$.

- A6.** Use the substitution $x+2 = 2 \tan \theta$ to obtain $\int \frac{1}{x^2 + 4x + 8} dx$.

- A7.** Prove by induction that $4^n - 1$ is divisible by 3 for all positive integers n .

[PTO]

- A8.** Express $\frac{x^2}{(x+1)^2}$ in the form $A + \frac{B}{x+1} + \frac{C}{(x+1)^2}$, ($x \neq -1$), stating the values of the constants A , B and C .

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A curve is defined by $y = \frac{x^2}{(x+1)^2}$, ($x \neq -1$).

- (i) Write down equations for its asymptotes. 2
- (ii) Find the stationary point and justify its nature. 4
- (iii) Sketch the curve showing clearly the features found in (i) and (ii). 2

- A9.** Functions $x(t)$ and $y(t)$ satisfy

$$\frac{dx}{dt} = -x^2y, \quad \frac{dy}{dt} = -xy^2.$$

When $t = 0$, $x = 1$ and $y = 2$.

- (a) Express $\frac{dy}{dx}$ in terms of x and y and hence obtain y as a function of x . 5
- (b) Deduce that $\frac{dx}{dt} = -2x^3$ and obtain x as a function of t for $t \geq 0$. 5