

Cfe AH Maths Homework (13) Complex Numbers

- ① Find the square roots of $5+12i$
- ② (a) Write $(1-i)$ in polar form.
(b) Hence evaluate $(1-i)^7$
- ③ Let $z = \cos\theta + i\sin\theta$.
 - (a) Use the binomial expansion to express z^3 in the form $a+ib$ where a and b are expressions involving $\sin\theta$ and $\cos\theta$.
 - (b) Use de Moivre's theorem to write down a second expression for z^3 .
 - (c) Using the results of (a) and (b) show that
$$\frac{\sin 3\theta}{\sin\theta} = p + q\sin^2\theta \quad \text{where } 0 < \theta < \frac{\pi}{2}$$
stating the values of p and q .
- ④ The equation $z^4 - 2z^3 + az^2 - 8z + 4 = 0$ has a solution for z which is wholly imaginary. Find the value of the real constant a .

- ⑤ Use de Moivre's theorem to find the cube roots of unity.
- ⑥ By using de Moivre's theorem, find the root of the equation $z^4 + 4 = 0$. Hence express $z^4 + 4$ as the product of two quadratic polynomials in z with real coefficients.
- ⑦ (a) Use de Moivre's theorem to show that
 $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
and $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.
- (b) Obtain an expression for $\tan 3\theta$ in terms of $\tan\theta$.
- ⑧ (a) Given that $-1 = \cos\theta + i\sin\theta$, $-\pi < \theta < \pi$, state the value of θ .
- (b) Use de Moivre's Theorem to find the non-real solutions z_1 and z_2 of the equation $z^3 + 1 = 0$
Hence show that $z_1^2 = -z_2$ and $z_2^2 = -z_1$
- (c) Plot all the solutions of $z^3 + 1 = 0$ on an Argand diagram.