Cfe AH Maths Homework (11)

① (a) Write down the binomial expansion of $(1+x)^5$

(b) Hence show that 0.9^5 is 0.59049 [2009]

② Use integration by parts to obtain the exact value of

$$\int_0^1 x \tan^{-1} x^2 \, dx$$

[2009]

③ The curve $y = x^{2x^2+1}$ is defined for $x > 0$.
Obtain the values of y and $\frac{dy}{dx}$ at the point where $x = 1$ [2009]

④ Obtain the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2$$

Given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = 1$ when $x = 0$, find the particular solution. [2008]

⑤ Express the complex number $z = -i + \frac{1}{1-i}$ in the form $z = x + iy$, stating the values of x and y. [2006]

⑥ Find $\int \frac{12x^3 - 6x}{x^4 - x^2 + 1} \, dx$ [2006]

⑦ Given that $(-1+2i)$ is a root of the equation

$$z^3 + 5z^2 + 11z + 15 = 0$$

obtain all the other roots. [2012]

⑧ Prove by induction that for all positive integers n.

$$\sum_{r=1}^{n} (4r^3 + 3r^2 + r) = n(n+1)^3$$

[2013]