

X100/13/01

NATIONAL
QUALIFICATIONS 2012

MONDAY, 21 MAY
1.00 PM – 4.00 PM

MATHEMATICS
ADVANCED HIGHER

Read carefully

- 1 Calculators may be used in this paper.
- 2 Candidates should answer **all** questions.
- 3 **Full credit will be given only where the solution contains appropriate working.**

Answer all the questions

1. (a) Given $f(x) = \frac{3x+1}{x^2+1}$, obtain $f'(x)$. 3
- (b) Let $g(x) = \cos^2 x \exp(\tan x)$. Obtain an expression for $g'(x)$ and simplify your answer. 4
2. The first and fourth terms of a geometric series are 2048 and 256 respectively. Calculate the value of the common ratio. 2
- Given that the sum of the first n terms is 4088, find the value of n . 3
3. Given that $(-1 + 2i)$ is a root of the equation

$$z^3 + 5z^2 + 11z + 15 = 0,$$
obtain all the roots. 4
Plot all the roots on an Argand diagram. 2
4. Write down and simplify the general term in the expansion of $\left(2x - \frac{1}{x^2}\right)^9$. 3
Hence, or otherwise, obtain the term independent of x . 2
5. Obtain an equation for the plane passing through the points $P(-2, 1, -1)$, $Q(1, 2, 3)$ and $R(3, 0, 1)$. 5
6. Write down the Maclaurin expansion of e^x as far as the term in x^3 . 1
Hence, or otherwise, obtain the Maclaurin expansion of $(1 + e^x)^2$ as far as the term in x^3 . 4
7. A function is defined by $f(x) = |x + 2|$ for all x .
(a) Sketch the graph of the function for $-3 \leq x \leq 3$. 2
(b) On a separate diagram, sketch the graph of $f'(x)$. 2
8. Use the substitution $x = 4 \sin \theta$ to evaluate $\int_0^2 \sqrt{16 - x^2} \, dx$. 6

9. A non-singular $n \times n$ matrix A satisfies the equation $A + A^{-1} = I$, where I is the $n \times n$ identity matrix. Show that $A^3 = kI$ and state the value of k . 4
10. Use the division algorithm to express 1234_{10} in base 7. 3
11. (a) Write down the derivative of $\sin^{-1}x$. 1
 (b) Use integration by parts to obtain $\int \sin^{-1}x \cdot \frac{x}{\sqrt{1-x^2}} dx$. 4
12. The radius of a cylindrical column of liquid is decreasing at the rate of 0.02 m s^{-1} , while the height is increasing at the rate of 0.01 m s^{-1} .
 Find the rate of change of the volume when the radius is 0.6 metres and the height is 2 metres. 5
[Recall that the volume of a cylinder is given by $V = \pi r^2 h$.]
13. A curve is defined parametrically, for all t , by the equations

$$x = 2t + \frac{1}{2}t^2, \quad y = \frac{1}{3}t^3 - 3t.$$

 Obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ as functions of t . 5
 Find the values of t at which the curve has stationary points and determine their nature. 3
 Show that the curve has exactly two points of inflexion. 2
14. (a) Use Gaussian elimination to obtain the solution of the following system of equations in terms of the parameter λ .

$$\begin{aligned} 4x + 6z &= 1 \\ 2x - 2y + 4z &= -1 \\ -x + y + \lambda z &= 2 \end{aligned}$$
 5
 (b) Describe what happens when $\lambda = -2$. 1
 (c) When $\lambda = -1.9$ the solution is $x = -22.25$, $y = 8.25$, $z = 15$.
 Find the solution when $\lambda = -2.1$. 2
 Comment on these solutions. 1

15. (a) Express $\frac{1}{(x-1)(x+2)^2}$ in partial fractions.

- (b) Obtain the general solution of the differential equation

$$(x-1)\frac{dy}{dx} - y = \frac{x-1}{(x+2)^2},$$

expressing your answer in the form $y = f(x)$.

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16. (a) Prove by induction that

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

for all integers $n \geq 1$.

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- (b) Show that the real part of $\frac{\left(\cos \frac{\pi}{18} + i \sin \frac{\pi}{18}\right)^{11}}{\left(\cos \frac{\pi}{36} + i \sin \frac{\pi}{36}\right)^4}$ is zero.

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[END OF QUESTION PAPER]