

X100/701

NATIONAL
QUALIFICATIONS
2009

THURSDAY, 21 MAY
1.00 PM – 4.00 PM

MATHEMATICS
ADVANCED HIGHER

Read carefully

1. Calculators may be used in this paper.
2. Candidates should answer all questions.
3. **Full credit will be given only where the solution contains appropriate working.**



Answer all the questions.

1. (a) Given $f(x) = (x+1)(x-2)^3$, obtain the values of x for which $f'(x) = 0$. 3
 (b) Calculate the gradient of the curve defined by $\frac{x^2}{y} + x = y - 5$ at the point $(3, -1)$. 4
2. Given the matrix $A = \begin{pmatrix} t+4 & 3t \\ 3 & 5 \end{pmatrix}$.
 (a) Find A^{-1} in terms of t when A is non-singular. 3
 (b) Write down the value of t such that A is singular. 1
 (c) Given that the transpose of A is $\begin{pmatrix} 6 & 3 \\ 6 & 5 \end{pmatrix}$, find t . 1
3. Given that

$$x^2 e^y \frac{dy}{dx} = 1$$

 and $y = 0$ when $x = 1$, find y in terms of x . 4
4. Prove by induction that, for all positive integers n ,

$$\sum_{r=1}^n \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}. \quad 5$$
5. Show that

$$\int_{\ln \frac{1}{2}}^{\ln 2} \frac{e^x + e^{-x}}{e^x - e^{-x}} dx = \ln \frac{9}{5}. \quad 4$$
6. Express $z = \frac{(1+2i)^2}{7-i}$ in the form $a+ib$ where a and b are real numbers.
 Show z on an Argand diagram and evaluate $|z|$ and $\arg(z)$. 6

7. Use the substitution $x = 2 \sin \theta$ to obtain the exact value of $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx$. 6
 (Note that $\cos 2A = 1 - 2 \sin^2 A$.)
8. (a) Write down the binomial expansion of $(1+x)^5$. 1
 (b) Hence show that 0.9^5 is 0.59049 . 2
9. Use integration by parts to obtain the exact value of $\int_0^1 x \tan^{-1} x^2 dx$. 5
10. Use the Euclidean algorithm to obtain the greatest common divisor of 1326 and 14654, expressing it in the form $1326a + 14654b$, where a and b are integers. 4
11. The curve $y = x^{2x^2+1}$ is defined for $x > 0$. Obtain the values of y and $\frac{dy}{dx}$ at the point where $x = 1$. 5
12. The first two terms of a geometric sequence are $a_1 = p$ and $a_2 = p^2$. Obtain expressions for S_n and S_{2n} in terms of p , where $S_k = \sum_{j=1}^k a_j$. 1,1
 Given that $S_{2n} = 65S_n$ show that $p^n = 64$. 2
 Given also that $a_3 = 2p$ and that $p > 0$, obtain the exact value of p and hence the value of n . 1,1
13. The function $f(x)$ is defined by

$$f(x) = \frac{x^2 + 2x}{x^2 - 1} \quad (x \neq \pm 1).$$

 Obtain equations for the asymptotes of the graph of $f(x)$. 3
 Show that $f(x)$ is a strictly decreasing function. 3
 Find the coordinates of the points where the graph of $f(x)$ crosses
 (i) the x -axis and 2
 (ii) the horizontal asymptote. 2
 Sketch the graph of $f(x)$, showing clearly all relevant features. 2

[Turn over for Questions 14 to 16 on Page four]

14. Express $\frac{x^2 + 6x - 4}{(x+2)^2(x-4)}$ in partial fractions. 4

Hence, or otherwise, obtain the first three non-zero terms in the Maclaurin expansion of $\frac{x^2 + 6x - 4}{(x+2)^2(x-4)}$. 5

15. (a) Solve the differential equation

$$(x+1) \frac{dy}{dx} - 3y = (x+1)^4$$

given that $y = 16$ when $x = 1$, expressing the answer in the form $y = f(x)$. 6

- (b) Hence find the area enclosed by the graphs of $y = f(x)$, $y = (1-x)^4$ and the x -axis. 4

16. (a) Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} x + y - z &= 6 \\ 2x - 3y + 2z &= 2 \\ -5x + 2y - 4z &= 1. \end{aligned}$$

- (b) Show that the line of intersection, L , of the planes $x + y - z = 6$ and $2x - 3y + 2z = 2$ has parametric equations 5

$$\begin{aligned} x &= \lambda \\ y &= +\lambda - 14 \\ z &= 5\lambda - 20. \end{aligned}$$

- (c) Find the acute angle between line L and the plane $-5x + 2y - 4z = 1$. 4

[END OF QUESTION PAPER]

Advanced Higher 2009

① (a) $f(x) = (x+1)(x-2)^3$

$$\begin{aligned}f'(x) &= (x+1) \cdot 3(x-2)^2 + (x-2)^3 \\&= (x-2)^2 (3x+3+x-2) \\&= (x-2)^2 (4x+1)\end{aligned}$$

$$f'(x) = 0 \Rightarrow (x-2)^2 (4x+1) = 0$$

$$\underline{x=2} \text{ or } \underline{x = -\frac{1}{4}}$$

② (b) $\frac{x^2}{y} + x = y - 5$

Differentiate.

$$\frac{y \cdot 2x - x^2 \cdot \frac{dy}{dx}}{y^2} + 1 = \frac{dy}{dx}$$

$$2xy - x^2 \frac{dy}{dx} + y^2 = y^2 \frac{dy}{dx}$$

$$2xy + y^2 = (x^2 + y^2) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy + y^2}$$

When $x=3$ and $y=-1$

$$\begin{aligned}\frac{dy}{dx} &= \frac{9+1}{-6+1} \\&= \frac{10}{-5} \\&= -2.\end{aligned}$$

gradient is -2 at $(3, -1)$

$$\textcircled{2} \quad A = \begin{pmatrix} t+4 & 3t \\ 3 & 5 \end{pmatrix}$$

$$\text{(a)} \quad A^{-1} = \frac{1}{5(t+4)-9t} \begin{pmatrix} 5 & -3t \\ -3 & t+4 \end{pmatrix}$$

$$= \frac{1}{20-4t} \begin{pmatrix} 5 & -3t \\ -3 & t+4 \end{pmatrix}$$

$$\text{(b)} \quad 20-4t = 0$$

$$t=5 \quad \text{For } A \text{ to be singular}$$

$$\text{(c)} \quad A' = \begin{pmatrix} 6 & 3 \\ 6 & 5 \end{pmatrix}$$

$$A = \begin{pmatrix} 6 & 6 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} t+4 & 3t \\ 3 & 5 \end{pmatrix}$$

$$\text{so } 6 = t+4$$

$$\underline{t=2}.$$

$$\textcircled{3} \quad x^2 e^y \frac{dy}{dx} = 1$$

$$\int e^y dy = \int \frac{dx}{x^2}$$

$$e^y = \frac{x^{-1}}{-1} + C.$$

$$e^y = -\frac{1}{x} + C.$$

$$\text{When } y=0, \quad x=1 \quad \Rightarrow \quad e^0 = -1+C$$

$$C=2$$

$$e^y = 2 - \frac{1}{x}$$

$$\underline{\underline{y = \ln 2 - \frac{1}{x}}}$$

$$\textcircled{4} \quad \sum_{r=1}^k \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}$$

When $n=1$

$$\begin{aligned} \text{LHS} &= \frac{1}{1(1+1)} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= 1 - \frac{1}{1+1} \\ &= \frac{1}{2}. \end{aligned}$$

So true for $n=1$

Assume the result is true for $n=k$.

$$\text{i.e. } \sum_{r=1}^k \frac{1}{r(r+1)} = 1 - \frac{1}{k+1}$$

Consider $n=k+1$

$$\begin{aligned} \sum_{r=1}^{k+1} \frac{1}{r(r+1)} &= \sum_{r=1}^k \frac{1}{r(r+1)} + \frac{1}{(k+1)(k+2)} \\ &= 1 - \frac{1}{k+1} + \frac{1}{(k+1)(k+2)} \\ &= 1 - \left[\frac{\frac{k+2}{k+1}}{(k+1)(k+2)} - \frac{1}{(k+1)(k+2)} \right] \\ &= 1 - \left[\frac{\frac{k+1}{k+1}}{(k+1)(k+2)} \right] \\ &= 1 - \frac{1}{k+2} \\ &= 1 - \frac{1}{(k+1)+1} \end{aligned}$$

So result is true for $n=k+1$.

Since result is true for $n=1$ and if result is true for $n=k$ then the result is true for $n=k+1$, then by induction the result is true for all positive integers n .

$$\textcircled{5} \quad \int_{\ln \frac{3}{2}}^{\ln 2} \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$$

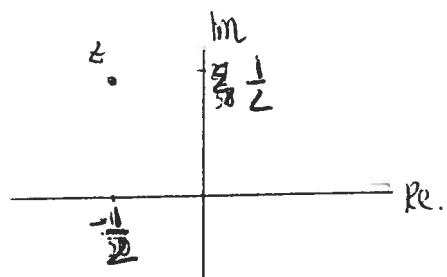
let $u = e^x - e^{-x}$
 $\frac{du}{dx} = e^x + e^{-x}$
 $du = (e^x + e^{-x}) dx$

limits $x = \ln \frac{3}{2}$ $u = e^{\ln \frac{3}{2}} - e^{-\ln \frac{3}{2}}$
 $= \frac{3}{2} - e^{\ln \frac{2}{3}}$
 $= \frac{3}{2} - \frac{2}{3}$
 $= \frac{9}{6} - \frac{4}{6}$
 $= \frac{5}{6},$

$x = \ln 2$ $u = e^{\ln 2} - e^{-\ln 2}$
 $= 2 - \frac{1}{2}$
 $= \frac{3}{2}$

$$\begin{aligned} & \int_{\ln \frac{3}{2}}^{\ln 2} \frac{e^x + e^{-x}}{e^x - e^{-x}} dx \\ &= \int_{\frac{5}{6}}^{\frac{3}{2}} \frac{du}{u} \\ &= [\ln |u|]_{\frac{5}{6}}^{\frac{3}{2}} \\ &= \ln \frac{3}{2} - \ln \frac{5}{6} \\ &= \ln \left(\frac{3}{2} : \frac{5}{6} \right) \\ &= \ln \left(\frac{3}{2} \times \frac{6}{5} \right) \\ &= \ln \frac{9}{5} \quad \text{as required.} \end{aligned}$$

$$\begin{aligned}
 \textcircled{6} \quad z &= \frac{(1+2i)^2}{(7-i)} \\
 z &= \frac{(1+2i)(7+i)}{(7-i)(7+i)} \\
 &= \frac{(1+4i-4)(7+i)}{50} \\
 &= \frac{(4i-3)(7+i)}{50} \\
 &= \frac{28i + 4i^2 - 21 - 3i}{50} \\
 &= \frac{25i - 25}{50} = -\frac{1}{2} + \frac{1}{2}i \\
 z &= -\frac{1}{2} + \frac{1}{2}i
 \end{aligned}$$



$$\begin{aligned}
 |z| &= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}} \\
 &= \underline{\sqrt{\frac{1}{2}}} = \frac{1}{2}\sqrt{2} \\
 \arg z &= \tan^{-1} \frac{\frac{1}{2}}{\frac{-1}{2}} = \tan^{-1}(-1) \\
 &= \tan^{-1}\left(-\frac{1}{1}\right) \\
 \arg z &= \underline{142.8^\circ} = 135^\circ
 \end{aligned}$$

180 - 67.8
=

$$\textcircled{7} \quad \int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx$$

$$\text{Let } x = 2\sin\theta.$$

$$\frac{dx}{d\theta} = 2\cos\theta$$

$$dx = 2\cos\theta d\theta$$

$$\text{limits } x=0 \rightarrow 2\sin\theta = 0 \\ \theta = 0$$

$$x=\sqrt{2} \quad 2\sin\theta = \sqrt{2} \\ \sin\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

$$\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{(2\sin\theta)^2}{\sqrt{4-(2\sin\theta)^2}} 2\cos\theta d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{4\sin^2\theta}{\sqrt{4-4\sin^2\theta}} \cdot 2\cos\theta d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{4\sin^2\theta}{\sqrt{4\cos^2\theta}} \cdot 2\cos\theta d\theta$$

$$= \int_0^{\frac{\pi}{4}} 4\sin^2\theta d\theta$$

$$2\sin^2 A = 1 - \cos 2A$$

$$4\sin^2\theta = 2 - 2\cos 2\theta$$

$$= \int_0^{\frac{\pi}{4}} (2 - 2\cos 2\theta) d\theta$$

$$= [2\theta - \sin 2\theta]_0^{\frac{\pi}{4}}$$

$$= \left(\frac{\pi}{2} - \sin \frac{\pi}{2} \right) - (0 - \sin 0)$$

$$= \frac{\pi}{2} - 1$$

③ (a) $(1+x)^5$

$$= 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$$

$$\begin{array}{ccccccccc} & & & & & 1 & & & \\ & & & & & | & & & \\ & & & & & 1 & 1 & & \\ & & & & & | & 2 & 1 & \\ & & & & & 1 & 3 & 3 & 1 \\ & & & & & | & 4 & 6 & 4 \\ & & & & & 1 & 5 & 10 & 10 & 5 & 1 \end{array}$$

(b) $19^5 = (1-0.1)^5$

$$= 1 + 5(-0.1) + 10(-0.1)^2 + 10(-0.1)^3 + 5(-0.1)^4 + (-0.1)^5$$

$$= 1 - 0.5 + 0.1 - 0.01 + 0.0005 - 0.00001$$

$$= 0.59049 \quad \text{as required}$$

$$\textcircled{9} \quad \int_0^1 x \tan^{-1} x^2 dx$$

$$= \left[\tan^{-1} x^2 \cdot \frac{x^2}{2} \right]_0^1 - \int_0^1 \frac{x^2}{2} \cdot \frac{1}{1+x^2} \cdot 2x dx$$

$$= \left[\frac{x^2 \tan^{-1} x^2}{2} \right]_0^1 - \int_0^1 \frac{x^3}{1+x^4} dx$$

$$= \left(\frac{1}{2} \tan^{-1} 1 - 0 \right) - \int_1^2 \frac{\frac{1}{4} du}{u}$$

$$= \frac{1}{2} \cdot \frac{\pi}{4} - \left[\frac{1}{4} \ln u \right]_1^2$$

$$= \frac{\pi}{8} - \frac{1}{4} \ln 2 + \frac{1}{4} \ln 1$$

$$= \frac{\pi}{8} - \frac{1}{4} \ln 2.$$

$$\begin{aligned} \text{let } u &= 1+x^4 \\ du &= 4x^3 dx \\ \frac{1}{4} du &= x^3 dx \end{aligned}$$

$$\begin{aligned} \text{limits } x=0 &\Rightarrow u=1 \\ x=1 &\Rightarrow u=2 \end{aligned}$$

$$\textcircled{10} \quad 14654 = 11 \times 1326 + 68$$

$$1326 = 19 \times 68 + 34$$

$$68 = 2 \times 34 + 0$$

The greatest common divisor is 34

$$34 = 1326 - 19 \times 68$$

$$= 1326 - 19(14654 - 11 \times 1326)$$

$$= 1326 - 19 \times 14654 + 209 \times 1326$$

$$= 210 \times 1326 - 19 \times 14654$$

$$\textcircled{11} \quad y = x^{2x^2+1}$$

When $x=1$ $y = 1^{2+1}$
 $y = 1$

$$y = x^{2x^2+1}$$

Take logs of both sides

$$\ln y = \ln x^{(2x^2+1)}$$

$$\ln y = (2x^2+1) \ln x$$

Differentiate

$$\frac{1}{y} \cdot \frac{dy}{dx} = (2x^2+1) \cdot \frac{1}{x} + \ln x \cdot (4x)$$

$$\frac{dy}{dx} = \frac{y}{x} (2x^2+1) + 4x^2 y \ln x$$

When $x=1, y=1$ so

$$\frac{dy}{dx} = \frac{1}{1} (2+1) + 4 \ln 1$$

$$\frac{dy}{dx} = 3.$$

$$\textcircled{12} \quad a = p \quad r = p.$$

Geometric

$$\begin{aligned} S_n &= \frac{a(1-r^n)}{(1-r)} \\ &= \frac{a(1-p^n)}{(1-p)} \end{aligned}$$

$$\begin{aligned} S_{2n} &= \frac{a(1-r^{2n})}{1-r} \\ &= \frac{a(1-p^{2n})}{(1-p)} \end{aligned}$$

$$S_{2n} = 65 S_n$$

$$\Rightarrow \frac{a \frac{(1-p^{2n})}{(1-p)}}{a \frac{(1-p^n)}{(1-p)}} = 65$$

$$(1-p^{2n}) = 65(1-p^n)$$

$$p^{2n} - 65p^n + 64 = 0$$

$$(p^n - 64)(p^n - 1) = 0$$

$$p^n = 64 \quad \text{or} \quad p^n = 1$$

$$\Rightarrow p = 1 \quad \text{not possible.}$$

$$\text{So } \underline{p^n = 64}. \quad \text{as required.}$$

$$a_3 = 2p \quad \text{but} \quad a_3 = p^3$$

$$\text{So } p^3 = 2p$$

$$p^3 - 2p = 0$$

$$p(p^2 - 2) = 0$$

$$p = 0 \quad \text{or} \quad p = \pm\sqrt[3]{2}.$$

$$\text{Since } p > 0 \quad p = \sqrt[3]{2}$$

$$\text{In } p^n = 64$$

$$(\sqrt[3]{2})^n = 64$$

$$2^{\frac{n}{3}} = 2^6$$

$$\frac{n}{3} = 6$$

$$\underline{n = 12}.$$

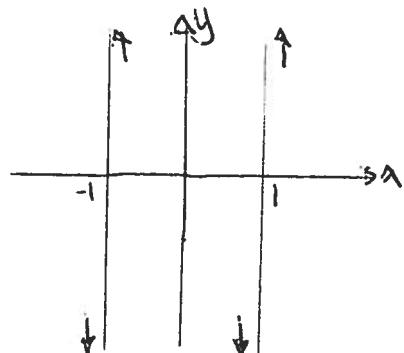
$$\begin{aligned} \textcircled{B} \quad f(x) &= \frac{x^2 + 2x}{x^2 - 1} \\ &= \frac{x(x+2)}{(x-1)(x+1)} \end{aligned}$$

Vertical asymptotes. $x^2 - 1 = 0$
 $\underline{x = \pm 1}$

Approach.

x	\rightarrow	-1	\rightarrow
$f(x)$	$\frac{(-)(+)}{(-)(-)}$	∞	$\frac{(-)(+)}{(-)(+)}$
	-	∞	+

x	\rightarrow	1	\rightarrow
$f(x)$	$\frac{(+)(+)}{(-)(+)}$	∞	$\frac{(+)(+)}{(+)(+)}$
	-	∞	+



Horizontal asymptotes.

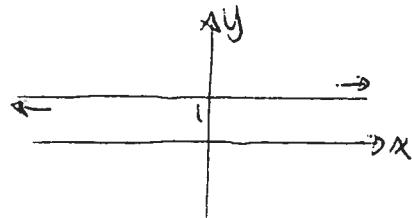
$$\begin{array}{c} 1 + \frac{2}{x} + \dots \\ x^2 - 1 \quad | \quad \frac{x^2 + 2x}{x^2 + 2x} \\ \hline x^2 \quad -1 \\ \hline 2x + 1 \end{array}$$

$$f(x) = 1 + \frac{2}{x} + \dots$$

As $x \rightarrow \infty$ $f(x) \rightarrow 1$ so $y = 1$ is a horizontal asymptote.

Approach As $x \rightarrow +\infty$ $f(x) \rightarrow 1 + (\text{a little bit})$ above.

As $x \rightarrow -\infty$ $f(x) \rightarrow 1 - (\text{a little bit})$ below.



$$f(x) = \frac{x^2 + 2x}{x^2 - 1}$$

$$\begin{aligned}
 f'(x) &= \frac{(x^2-1)(2x+2) - (x^2+2x)(2x)}{(x^2-1)^2} \\
 &= \frac{2x^3 + 2x^2 - 2x - 2 - 2x^3 - 4x^2}{(x^2-1)^2} \\
 &= \frac{-2x^2 - 2x - 2}{(x^2-1)^2} \\
 &= \frac{-2(x^2+x+1)}{(x^2-1)^2}
 \end{aligned}$$

So $f'(x)$ is always negative
 $\Rightarrow f(x)$ is always decreasing.

$$\text{Consider } y = x^2 + x + 1.$$

$$\begin{aligned}
 &\Delta^2 - 4ac \\
 &= 1 - 4 \times 1 \times 1 \\
 &= -3 \quad \text{doesn't cut } x\text{-axis} \\
 &\Rightarrow \text{always positive } (> 0)
 \end{aligned}$$

Cuts x -axis put $y=0$.

$$\frac{x^2 + 2x}{x^2 - 1} = 0$$

$$x^2 + 2x = 0$$

$$x(x+2) = 0$$

$$x=0, x=-2$$

$$(0,0) (-2,0)$$

Cuts horizontal asymptote.

$$\frac{x^2+2x}{x^2-1} = 1$$

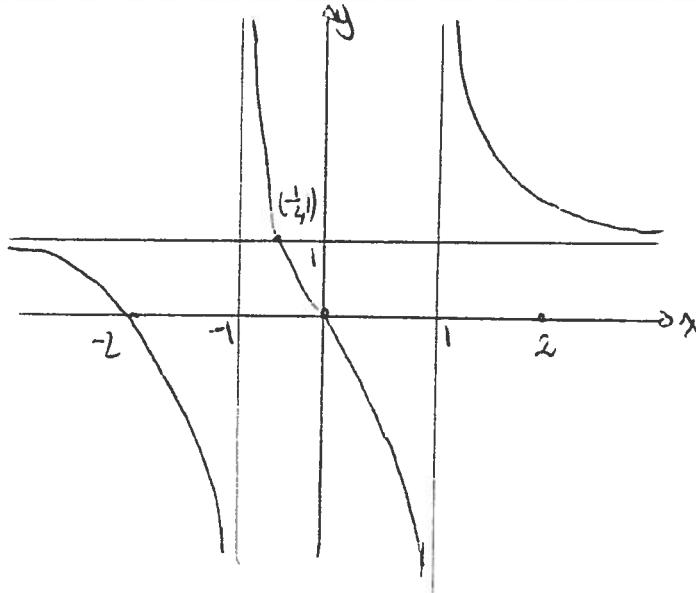
$$x^2+2x = x^2-1$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$\begin{aligned}
 \Rightarrow y &= \frac{\frac{1}{4}}{\frac{1}{4}-1} + (-1) \\
 &= \left(\frac{-3}{4}\right) \div \left(\frac{-3}{4}\right) \\
 &= 1
 \end{aligned}$$

$$\left(-\frac{1}{2}, 1\right)$$



(14)

$$\begin{aligned}
 & \frac{x^2 + 6x - 4}{(x+2)^2(x-4)} \\
 &= \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x-4)} \\
 &= \frac{A(x+2)(x-4) + B(x-4) + C(x+2)^2}{(x+2)^2(x-4)}
 \end{aligned}$$

$$\text{So } x^2 + 6x - 4 = A(x+2)(x-4) + B(x-4) + C(x+2)^2(x-4)$$

$$\begin{aligned}
 \text{Let } x = 4 & \quad 16 + 24 - 4 = 36C \\
 & \quad C = 1
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } x = -2 & \quad 4 - 12 - 4 = -6B \\
 & \quad -12 = -6B \\
 & \quad B = 2
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } x = 0 & \quad -4 = -8A - 4B + 4C \\
 & \quad -4 = -8A - 8 + 4 \\
 & \quad A = 0
 \end{aligned}$$

$$\text{So } \frac{x^2 + 6x - 4}{(x+2)^2(x-4)} = \frac{2}{(x+2)^2} + \frac{1}{(x-4)}$$

$$f(x) = 2(x+2)^{-2} + (x-4)^{-1}$$

$$f(0) = \frac{2}{4} + \frac{1}{-4} \\ = -\frac{1}{4}$$

$$f'(x) = -4(x+2)^{-3} - (x-4)^{-2}$$

$$= -\frac{4}{(x+2)^3} - \frac{1}{(x-4)^2}$$

$$f'(0) = -\frac{4}{3} + \frac{1}{4} \cdot \frac{4}{8} - \frac{1}{16}$$

$$= \frac{1}{4} - \frac{9}{16}$$

$$f''(x) = 12(x+2)^{-4} + 2(x-4)^{-3}$$

$$= \frac{12}{(x+2)^4} + \frac{2}{(x-4)^3}$$

$$f''(0) = \frac{12}{16} + \frac{2}{(-4)^3}$$

$$= \frac{23}{32}$$

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \dots$$

$$= \frac{1}{4} + x \left(-\frac{9}{16} \right) + \frac{x^2}{2} \cdot \frac{23}{32} + \dots$$

$$= \frac{1}{4} - \frac{9x}{16} + \frac{x^2}{64} + \dots$$

$$(15) (a) (x+1) \frac{dy}{dx} - 3y = (x+1)^4$$

$$\frac{dy}{dx} - \frac{3}{x+1} y = (x+1)^3$$

Integrating Factor

$$\begin{aligned} & e^{\int (-\frac{3}{x+1}) dx} \\ &= e^{-3 \ln|x+1|} \\ &= e^{\ln|x+1|^{-3}} \\ &= \frac{1}{(x+1)^3} \end{aligned}$$

$$\frac{d}{dx} \left(\frac{1}{(x+1)^3} y \right) = (x+1)^3 \cdot \frac{1}{(x+1)^3}$$

$$\frac{1}{(x+1)^3} y = \int 1 dx$$

$$\left(\frac{1}{x+1}\right)^3 y = x + C.$$

$$\text{When } x=1, y=16$$

$$\frac{1}{2^3} \cdot 16 = 1+C$$

$$2 = 1+C$$

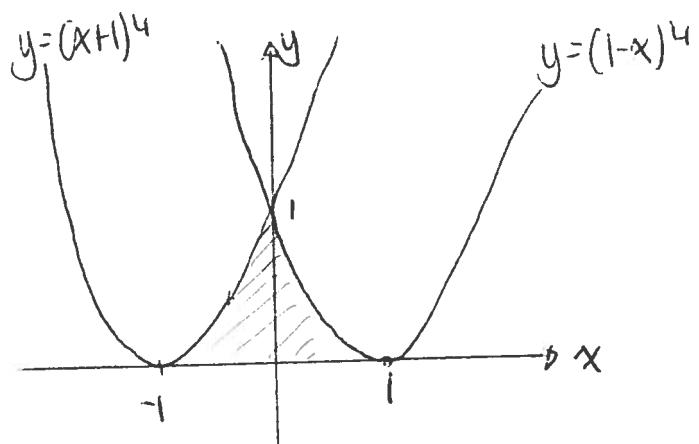
$$C = 1$$

$$\left(\frac{1}{x+1}\right)^3 y = x+1$$

$$y = (x+1)^4.$$

$$(b) \quad y = (x+1)^4$$

$$\begin{aligned} y &= (1-x)^4 \\ &= (-x+1)^4 \\ &= (x-1)^4 \end{aligned}$$



$$\text{area} = 2 \int_0^1 (1-x)^4 dx$$

$$= 2 \left[\frac{(1-x)^5}{5} \Big|_0^1 \right]$$

$$= 2 \left[-\frac{1}{5} (1-x)^5 \Big|_0^1 \right]$$

$$= 2 \left(-\frac{1}{5}(0) - \left(-\frac{2}{5} \right) \right)$$

$$= 2 \left(\frac{2}{5} \right)$$

$$= \frac{4}{5}$$

$$\text{area} = \frac{4}{5} \text{ square units}$$

$$(16) \text{ (a)} \quad x + y - z = 6$$

$$2x - 3y + 2z = 2$$

$$-5x + 2y - 4z = 1$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 2 & 3 & 2 & 2 \\ -5 & 2 & -4 & 1 \end{array} \right)$$

$$\begin{array}{l} R2 - 2R1 \\ R3 + 5R1 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -5 & 4 & -10 \\ 0 & 7 & -9 & 31 \end{array} \right)$$

$$\begin{array}{l} \\ \\ 7R2 + 5R3 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -5 & 4 & -10 \\ 0 & 0 & -17 & 85 \end{array} \right)$$

$$-17z = 85$$

$$z = -5$$

$$-5y + 4z = -10$$

$$-5y = 10$$

$$y = -2$$

$$x + y - z = 6$$

$$x - 2 + 5 = 6$$

$$x = 3$$

$$x = 3, y = -2, z = -5$$

$$\begin{aligned}
 (b) \quad \underline{d} &= \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & -3 & 2 \end{vmatrix} \\
 &= \underline{i}(2-3) - \underline{j}(2+2) + \underline{k}(-3-2) \\
 &= -\underline{i} - 4\underline{j} - 5\underline{k}.
 \end{aligned}$$

direction vector $\underline{i} + 4\underline{j} + 5\underline{k}$.

$$\text{let } x=0 \quad y-z=6 \quad \dots \quad (1)$$

$$-3y+2z=2 \quad \dots \quad (2)$$

$$(2)-(1) \quad -y = 14$$

$$y = -14 \quad z = -20$$

$$\text{Line.} \quad \frac{x-0}{1} = \frac{y+14}{4} = \frac{z+20}{5} \quad (= \lambda)$$

$$x = \lambda$$

$$y = 4\lambda - 14$$

$$z = 5\lambda - 20$$

$$(c) \quad \text{Angle between} \quad \underline{d} = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} \quad \text{and} \quad \underline{n} = \begin{pmatrix} -5 \\ 2 \\ -4 \end{pmatrix}$$

$$\underline{d} \cdot \underline{n} = -5 + 8 - 20 \\ = -17$$

$$|\underline{d}| = \sqrt{1+16+25} \\ = \sqrt{42} \quad |\underline{n}| = \sqrt{25+16+16} \\ = \sqrt{45}$$

$$\cos \theta = \frac{-17}{\sqrt{42} \sqrt{45}}$$

$$\theta = 113^\circ$$

$$\text{acute angle} = 180^\circ - 113^\circ \\ = 67^\circ$$

angle between line and plane
= $90 - 67$
= 23°